

# PHY 252 Lab 7: The Emission Spectrum of Hydrogen

Spring 2008

## 1 Introduction

In this experiment you will measure the visible part of the hydrogen spectrum, the Balmer series, and determine the Rydberg constant  $R_y$ .

If parallel rays of light are incident perpendicularly to the plane of a diffraction grating, with uniform phase over the grating, then one will observe a diffraction pattern which will have a series of intensity maxima at angles  $\theta$  satisfying the equation

$$n\lambda = d \sin \theta, \quad n = 0, 1, 2, \dots \quad (1)$$

where  $\lambda$  is the wavelength of the emitted light, and  $d$  is spacing between adjacent grooves on the grating.

Thus the grating will allow a determination of an unknown wavelength if the positions of the intensity maxima are measured and  $d$  is known.

## 2 Apparatus

The means for mounting the grating, obtaining parallel rays and measuring the positions of the maxima are provided by the spectrometer table, indicated in the Figure. Light from the source passes through a collimator which consists of a vertical slit and lens. With proper adjustment the collimator can be made to emit parallel rays. This light passes through the grating and is observed in a telescope which has been focused to accept parallel refracted rays. The positions of the various intensity maxima are then determined by noting their position as angles on the vernier scale on the spectrometer table.

## 3 Alignment

It is extremely crucial for the success of this experiment that the spectrometer is aligned accurately. Your TA will show you how to do this. The steps are the following:

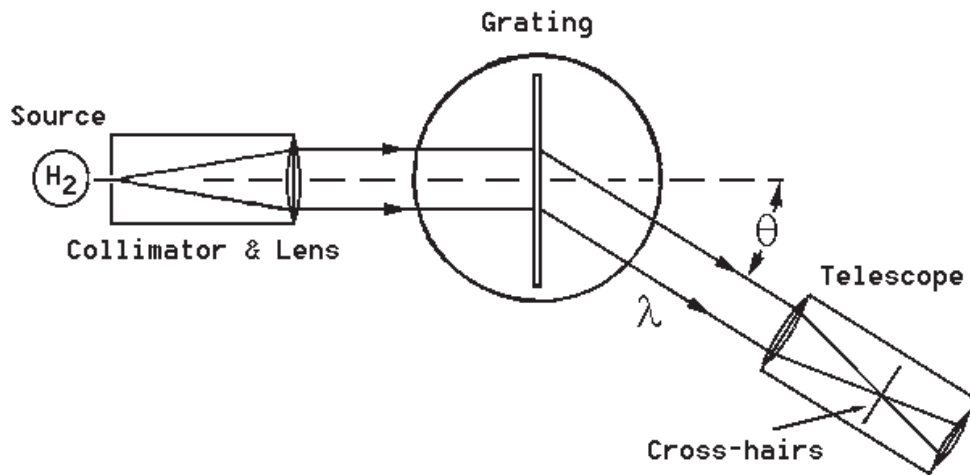


Figure 1: Spectrometer setup.

1. **Adjustment of the telescope:** Adjust the telescope so that it focuses parallel rays, by observing some distant object. This includes adjusting the eyepiece to insure that the image of the distant object falls right on the cross hairs, with no evident parallax.
2. **Adjustment of collimator to emit parallel rays:** Move the telescope until you are looking directly into the collimator. Adjust the collimator until you obtain a sharp image of the collimator slit.
3. **Adjustment of the grating table:** Place a mirror in the position of grating. Introduce some light into the hole on the telescope barrel. You should see in addition to the cross hairs also a faint image of the cross hairs due to the light being reflected from the mirror. Adjust the grating table until the two images overlap.

## 4 Measurement

The spectrometer is now ready to measure wavelengths; make sure you understand how to read the vernier scale for the angles. Place hydrogen source so that the light falls into the collimator. Insert the grating in its slot after reading off the spacing  $d$ . Now slowly change the angle setting of the telescope until you see a line. Read the angle. Continue until you see the next line (it will have a different colour) and so on. You should see three different colors. The first one is actually most difficult to see. If you rotate still further you will see the same sequence of lines appearing again, but much fainter. This is the second order pattern ( $n = 2$ ). Try to see as many lines and orders as possible. Repeat this sequence of measurements on the *opposite* side of the spectrometer. This gives better precision in the measured angles: for each line and order you have now a pair of measured angles  $\theta_L$  and  $\theta_R$ , from which you can find  $\theta = (\theta_L - \theta_R)/2$ .

## 5 Analysis

In the experiment, you have several diffraction angles  $\theta$  for emission lines from an excited hydrogen gas. You know that for the Bohr model photons should be emitted with energies corresponding to energy differences between initial  $i$  and final  $f$  states or

$$\Delta E = E_i - E_f = \frac{-E_0}{n_i^2} - \frac{-E_0}{n_f^2} = E_0 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \frac{hc}{\lambda}. \quad (2)$$

You also know that the diffraction angles should be given by

$$n_d \lambda = d \sin \theta \quad (3)$$

where  $n_d$  is the integer describing the diffraction order. Usually you can only see three spectral lines in both positive ( $n_d = +1$ ) and negative ( $n_d = -1$ ) first diffraction orders, and maybe also in second orders—but the angles at positive and negative orders, and at first order and second order, are all related to each other. Therefore if you can measure just three lines (perhaps red, blue-green, and violet), you have only three *independent* measurements. At the same time, ideally you'd like to use your data to determine  $E_0$ ,  $d$ ,  $n_i$ , and  $n_f$  for each of your lines but of course you can't determine four unknowns from three knowns!

What to do? Well, let's start by assuming that we know a rough value for  $E_0 = 13.6$  eV and asking about what wavelengths we might have a chance of seeing:

$n_f = 1$			$n_f = 2$			$n_f = 3$		
$n_i$	$\Delta E$ (eV)	$\lambda$ (nm)	$n_i$	$\Delta E$ (eV)	$\lambda$ (nm)	$n_i$	$\Delta E$ (eV)	$\lambda$ (nm)
4	12.75	97.3	5	2.86	434.2	6	1.13	1094
3	12.09	102.6	4	2.55	486.3	5	0.97	1282
2	10.20	121.6	3	1.89	656.5	4	0.66	1876

Since your eye can only see in the range  $\lambda \simeq 400\text{--}700$  nm, we can pretty safely assume that you're seeing states with  $n_f = 2$  and  $n_i = 5, 4$ , and  $3$ . If we were to indeed assume that the Bohr model works and that we have these values for the principal quantum numbers  $n_i$  and  $n_f$ , then we can put Eqs. 2 and 3 together to obtain

$$\lambda = \frac{hc}{E_0} \frac{1}{1/n_f^2 - 1/n_i^2} = \frac{d \sin \theta}{n_d} \quad (4)$$

$$y \equiv n_d \frac{hc}{E_0} \frac{1}{d} = \sin \theta \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

What varies between our different measurements? Just  $n_i$  and  $\sin \theta$ . Therefore in Eq. 4 we have put everything that remains constant on the left, and things that vary on the right. Therefore if we have three different values of  $\theta$ , and three different values of  $n_i$ , and put those into the right side of Eq. 4, we should ideally get the same value ( $y$ ) on the left side in each case. A logical strategy then is to obtain an average value  $\bar{y}$  from the three calculations of  $\sin \theta (1/n_f^2 - 1/n_i^2)$  for the three values of  $\theta$  and the three values of  $n_i$ .

With that experimental value of  $\bar{y}$ , we can solve for only one of the variables  $n_d$ ,  $h$ ,  $c$ ,  $E_0$ , and  $d$  out of the result of

$$\bar{y} = n_d \frac{hc}{E_0} \frac{1}{d}.$$

Now if we were Henry Rowland in the late 1800s at Harvard University, we might make a precision grating ruling engine based on measuring the thread pitch (such as in teeth per cm) of a screw against a precision meter stick, and use that plus measured gear ratios to cut scratches in glass at precise intervals, and thus have a grating with known  $d$ . In our case, I think it's more honest to say that we know  $E_0 = 13.606$  eV and  $hc = 1239.8$  eV·nm, and determine  $d$  from

$$d = n_d \frac{hc}{E_0} \frac{1}{\bar{y}}. \quad (5)$$

This is perhaps not as satisfying as determining  $E_0$  from first principles, but it's well matched to an honest analysis of your experiment. Also, since you have three values of  $y$ , you can calculate the error on  $\bar{y}$  from

$$\sigma_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{N - 2}} \quad (6)$$

so that you can give a value of  $y = \bar{y} \pm \sigma_y$  and thus get an error on your determination of  $d$  as well.