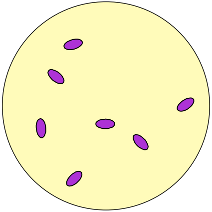
Thomson's plum pudding model

- Atoms, like nearly all matter, are electrically neutral.
- "Corpuscles" of negative charge have known e/m ratio. Since remaining bit of hydrogen atom is equally positively charged, and mass of atom could be estimated, Thomson inferred that electrons have equal but opposite charge and thus carry only $1/800^{\text{th}}$ of the mass.



- Therefore bulk of atom must be positive with little negative bits inside: the plum pudding model.
- Post-Einstein: "classical" radius of the electron $r_e = 2.82 \times 10^{-15}$ meters. This equates energy required to shrink an electron charge from a radius of infinity down to a radius at which electrostatic energy stored is equal to mc^2 rest energy. Compare with Einstein's Brownian motion estimate of atom sizes of about 2×10^{-10} meters.

Elementary charge

- R.A. Millikan and H. Fletcher at the University of Chicago, 1909: Charge always comes in integer multiples of a basic value. So now we really have e pinned down, and therefore we also know m_e!
- Millikan also carried out further studies of the photoelectric effect, confirming Einstein's prediction and obtaining an improved value for Planck's constant h during the time period 1912–1915.

R.A. Millikan (1868–1953; Nobel Prize 1923)

Indigestion from plum pudding

Displacing an electron by r from the center of an atom of radius R and positive charge Z gives a restoring force of

$$F = \frac{Ze^2}{4\pi\epsilon_0 R^3} r \qquad (1)$$

(see Krane Eq. 6.1) which has the form $F = kr$ with $k \equiv Ze^2/(4\pi\epsilon_0 R^3)$. Therefore for hydrogen ($Z = 1$) we have a harmonic oscillator with a resonant frequency of

$$\begin{aligned} \nu &= \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\frac{e}{\sqrt{4\pi\epsilon_0\, R^3\, m_e}} \\ &= \frac{1}{2\pi}\frac{1.602 \times 10^{-19}}{\sqrt{4\pi \cdot 8.854 \times 10^{-12} \cdot (1 \times 10^{-10})^3 \cdot 9.109 \times 10^{-31}}} \\ &= 2.5 \times 10^{15}\ \text{Hz} \end{aligned}$$

This corresponds to a radiation wavelength of $\lambda = c/\nu = 120$ nm.

Meanwhile. . . Radioactivity!

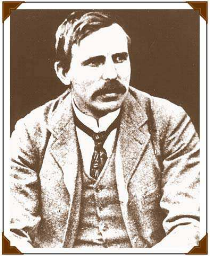
- Early 1896: Henri Becquerel noticed that uranium compounds would fog photographic plates–the discovery of radioactivity.
- 1898: Marie Sklodowska Curie measures radioactivity by looking at ionization of air. Unaffected by chemical binding, heat, etc.! Husband Pierre then joins research; they discover radium and polonium.
- Radioactive decay releases energies in the MeV range!!!
- Becquerel, and Marie and Pierre Curie share the 1903 Nobel Prize in Physics. Marie is awarded the 1911 Nobel Prize in Chemistry.

Antoine Henri Becquerel (1852–1908)

Marie Curie (1867–1934) and Pierre Curie

Enter Rutherford: the alpha male

- Grew up on a farm in New Zealand, and studied at the University there. Applied for a graduate scholarship at Cambridge and worked at home while awaiting a reply. When the scholarship letter came (1894), he threw down his shovel and said "That's the last potato I will ever dig."
- McGill University in Montreal, 1898–1907. University of Manchester, 1907–1919. Cavendish Professor at Cambridge, 1919–1937.
- "All science is either physics or stamp collecting"



Ernest Rutherford (1871–1937; Nobel Prize in Chemistry, 1908)

Discovery of the alpha particle

Rutherford at McGill in Montreal, 1905:

- Fill thin-glass-wall tube with radon, which emits alpha particles (α).
- Surround that tube with another thick-walled, evacuated tube.
- After a few days, helium identified in the outer tube by its characteristic spectrum.
- Alpha particles are helium nuclei (2 protons, 2 neutrons, or ${}^4_2\mathrm{He}_2$, using ${}^{\mathrm{A}}_{\mathrm{n}}\mathrm{X}_{\mathrm{Z}}$).
- Other radioactive decay emissions: β^- are electrons, γ are very energetic photons.

1908 Nobel Prize in Chemistry.

Use α particles to probe the nucleus

- Radium emits α particles with 5 MeV kinetic energy. Assume for now (not quite right) that $m = 2 \cdot 938.3 + 2 \cdot 939.6 = 3757.8\ \mathrm{MeV/c^2}$; gives $v = 2\sqrt{5\ \mathrm{MeV}/(3757.8\ \mathrm{MeV/c^2})} = 0.365\,c = 2.2 \times 10^7$ m/s.
- Equivalently, $m = 2 \cdot 1.673 \times 10^{-27} + 2 \cdot 1.675 \times 10^{-27} = 6.696 \times 10^{-27}$ kg and $v = 2\sqrt{(5 \times 10^6\ \mathrm{eV}) \cdot (1.602 \times 10^{-19}\ \mathrm{J/eV})/(6.696 \times 10^{-27}\ \mathrm{kg})} = 2.2 \times 10^7$ m/s.
- Gold nucleus significantly outweighs α. Gold can be hammered into very thin foils. Interatomic spacing:

$$\left(\frac{A}{\rho N_A}\right)^{1/3} = \left(\frac{197\ \mathrm{g/mol}}{(18.9\ \mathrm{g/cm^3}) \cdot (6.02 \times 10^{23}\ \mathrm{mol^{-1}})}\right)^{1/3} = 2.6 \times 10^{-8}\ \mathrm{cm}$$

α backscattering?

- So α particles have $\beta \simeq 0.07$ and mass of about $4/197 \simeq 0.020$ that of gold nuclei.
- While at McGill, Rutherford had noticed significant scattering at large angles. That seemed odd. . .
- After receiving Nobel Prize, Rutherford is offered position at Manchester in U.K. Assigns further investigation of this to Ernest Marsden (18 year old undergrad at start of experiments), aided by Rutherford's "postdoc" Hans Geiger.

Thomson scattering I

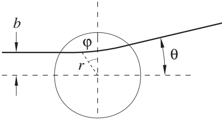
- Particle of charge ze approaches a sphere of charge Ze in a radius R, with an impact parameter (distance off from the centerline) of b.
- For small θ, distance particle travels through sphere is chord c at b, or

$$b^2 + \left(\frac{c}{2}\right)^2 = R^2 \qquad \text{or} \qquad c = 2\sqrt{R^2 - b^2}.$$

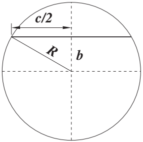
- Time that particle spends inside the sphere is (Krane Eq. 6.5)

$$T = 2\frac{\sqrt{R^2 - b^2}}{v}$$

Scattering geometry (like Krane Fig. 6.3):



Chord of sphere:



Thomson scattering II

- What force does particle experience? Use Eq. 1:

$$F_y = F\cos\varphi \simeq kzr\frac{b}{r} \simeq kzb \qquad \text{with} \qquad k \equiv \frac{Ze^2}{4\pi\epsilon_0 R^3}$$

- Momentum impulse transferred to the scattered particle (like Krane Eqs. 6.2, 6.6):

$$\Delta p_y = \int F_y\,dt = \int kzb\,dt = kzbT = \frac{2kzb}{v}\sqrt{R^2 - b^2}.$$

- Scattering angle θ is then (Krane Eq. 6.8)

$$\theta \simeq \frac{\Delta p_y}{p} = \frac{2kzb}{mv^2}\sqrt{R^2 - b^2}. \qquad (2)$$

- For $b = R/2$ and using Eq. 1, we have (like Krane Eq. 6.9)

$$\theta_{\text{typ.}} = 2\,k\,\frac{z(R/2)}{E_k}\,R\sqrt{\frac{3}{4}} = \sqrt{3}\,\frac{Ze^2}{4\pi\epsilon_0 R^3}\,\frac{zR^2}{2E_k} = \frac{\sqrt{3}\,zZe^2}{8\pi\epsilon_0\,R\,E_k} \qquad (3)$$

Thomson scattering III

- Use $z = 2$, $Z = 79$, $R = 2.6 \times 10^{-10}$ m, $E_k = (5 \times 10^6\ \text{eV}) \cdot (1.602 \times 10^{-19}\ \text{J/eV}) = 8.0 \times 10^{-13}$ J to obtain

$$\begin{aligned} \theta_{\text{typ.}} &= \frac{\sqrt{3}\, zZe^2}{8\pi\epsilon_0\, R\, E_k} \\ &= \frac{\sqrt{3} \cdot 2 \cdot 79 \cdot (1.602 \times 10^{-19})^2}{8\pi \cdot (8.854 \times 10^{-12}) \cdot (2.6 \times 10^{-10}) \cdot (8.0 \times 10^{-13})} \\ &= 1.5 \times 10^{-4}\ \text{rad} = 0.15\ \text{mrad} \qquad (4) \end{aligned}$$

Rutherford's suggestion

- So. . . What's inside the bowl of plum pudding? Let's shoot bullets of $\sim 8000\times$ mass of electron to find out!
- Gold hammered to 2 μm thickness: about 10^4 atoms thick.
- Net deflection angle for N uncorrelated scatterings of value θ each is $\sqrt{N}\theta$
- Thus for gold foil we expect about $100\theta_{\text{typ.}}$, or about 15 mrad.
- Not at all consistent with Rutherford's earlier observations! Look into it more closely.
- Rutherford assigns experiment to Ernest Marsden (age 20) and Hans Geiger. They use a microscope focused on a ZnS screen to observe flashes of light from single α particles.

The unexpected

Marsden and Geiger find significant scattering at large angles!
Rutherford:

> *It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration I realised that this scattering backwards must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greatest part of the mass of the atom was concentrated in a minute nucleus.*

As reported by Rhodes in *The making of the atomic bomb*

Rutherford's interpretation

- Return to Eq. 3 of

$$\theta_{\text{typ.}} = \frac{\sqrt{3}\, zZe^2}{8\pi\epsilon_0\, R\, E_k}$$

- We need to scale up from single scattering of 0.15 mrad to 1.5 rad, or a factor of 10^4.
- What's the chance of having *all* 10^4 scattering events give $\theta > \theta_{\text{typ.}}$? Assume probability of (1/2) for each scatter to be larger than $\theta_{\text{typ.}}$; net probability is then $(1/2)^{10^4}$. If we have $A = B^C$, then

$$\log_{10} A = \log_{10} B^C = C \log_{10} B,$$

so $(1/2)^{10^4} = 10^{10^4 \log_{10}(1/2)} = 10^{-3000}$.
- To do this by N uncorrelated scatters, we would need $\sqrt{N} = 10^4$ or $N = 10^8$ or a foil thickness of $10^8 \cdot 2.6 \times 10^{-10}$ m $= 2.6$ cm. Ludicrous!
- What can we change? R is the only thing! Must scale it down by 10^{-4}, from 2.6×10^{-10} to 2.6×10^{-14}.

Re-examine the situation

- Conclusion: the nucleus is *very* small. When Rutherford figures it out (early 1911), he marches into Geiger's office humming "Onward Christian Soldiers" and announces, "I know what the atom looks like!"
- With a point-like nucleus, electrons must be in some sort of orbital motion.
- Now we re-think the problem, with $R \ll b$. It can be shown that a particle follows a hyperbolic path when it passes near a point source of a $1/r^2$ repulsive force. For Coulomb repulsion, one obtains (Krane Eq. 6.12)

$$\frac{1}{r} = \frac{1}{b}\sin\varphi + \frac{zZe^2}{8\pi\epsilon_0 b^2 E_k}(\cos\varphi - 1), \tag{5}$$

  where E_k is the kinetic energy of the alpha particle.
- For scattering angles of $\theta = \pi - \varphi$, one can solve for b to obtain (Krane Eq. 6.13)

$$b = \frac{zZe^2}{8\pi\epsilon_0 E_k}\cot(\frac{1}{2}\theta). \tag{6}$$

Rutherford scattering II

- Fraction f of scatterings at an angle of θ or larger will be given by the fractional area taken up by atoms with impact parameter b in a foil of thickness t; this can be found to be (Krane Eq. 6.15)

$$f_{>\theta} = \frac{\rho N_A}{A}\, t\, \pi b^2. \tag{7}$$

- To find the range of impact parameters b to $b + db$ which produce scattering within an angular range $d\theta$, we must find

$$d(f_{>\theta}) = \frac{N_A \rho}{A}\, t\, (2\pi b\, db). \tag{8}$$

- The db term can be found from Eq. 6 to be (Krane Eq. 6.16)

$$db = \frac{zZe^2}{8\pi\epsilon_0 E_k}\, (-\csc^2[\frac{1}{2}\theta])(\frac{1}{2}d\theta), \tag{9}$$

which gives (Krane Eq. 6.17)

$$d(f_{>\theta}) = \frac{N_A \rho}{A}\, \pi t \left(\frac{zZe^2}{8\pi\epsilon_0 E_k}\right)^2 \csc^2(\frac{1}{2}\theta)\cot(\frac{1}{2}\theta)\, d\theta. \tag{10}$$

Rutherford scattering III

- In fact, our detector is not likely to collect *all* events at an angle larger than θ; instead, we will consider the fraction of events which fall in a ring at a distance r from the scatterer which collects an angular range of $d\theta$ about θ.
- The detector area integrated over 2π azimuthally, the radius of the $d\theta$ ring is $r \sin\theta$, and the width of the ring is $r\,d\theta$, so the detector area is $2\pi r^2 \sin\theta\, d\theta$.
- The fraction $N(\theta)$ of events we expect to detect can then be shown to be (Krane Eq. 6.18)

$$N(\theta) = \frac{N_A \rho}{A} \frac{t}{4r^2} \left(\frac{zZe^2}{8\pi\epsilon_0 E_k} \right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}. \tag{11}$$

Angular distribution

Agreement between Rutherford's theory and Marsden's experiments (Krane Fig. 6.14):

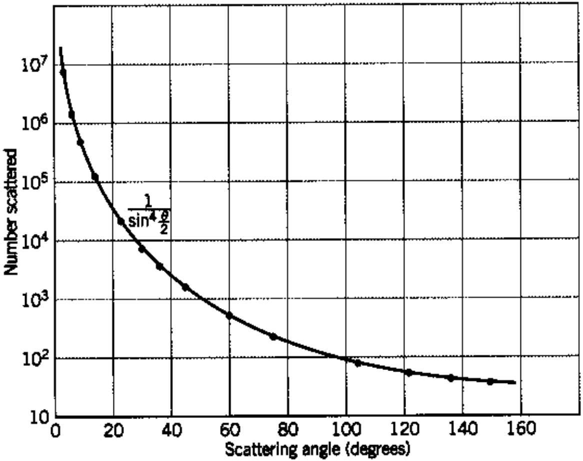


FIGURE 6.14 The dependence of scattering rate on the scattering angle θ, using a gold foil. The $\sin^{-4}(\theta/2)$ dependence is exactly as predicted by the Rutherford formula.

Effect of foil thickness

Rutherford predicts that $N(\theta) \propto t$, the foil thickness, whereas recall that the Thomson model would have predicted $N(\theta) \propto \sqrt{t}$. See Krane Fig. 6.11:

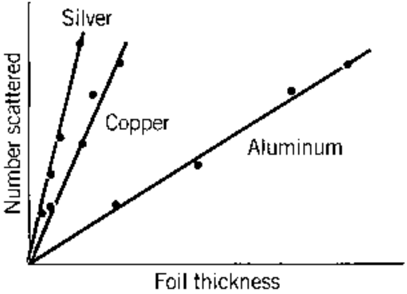


FIGURE 6.11 The dependence of scattering rate on foil thickness for three different scattering foils.

Effect of atomic number Z

Rutherford predicts $N(\theta) \propto Z^2$, which stands in contrast to the linear dependence on Z in Eq. 3. See Krane Fig. 6.12:

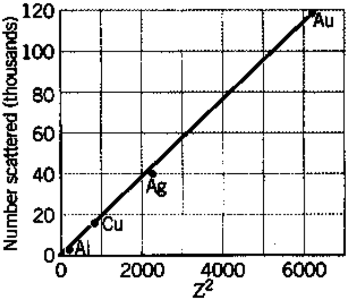


FIGURE 6.12 The dependence of scattering rate on the nuclear charge Z for foils of different materials. The data are plotted against Z^2.

Distance of closest approach

- In backscattering, the kinetic energy of the alpha particle must be converted completely into electrostatic potential energy at the point of closest approach.
- This gives a minimum radius of (Krane Eq. 6.22)

$$r_{\text{min}} = \frac{1}{4\pi\epsilon_0}\frac{zZe^2}{E_k}, \tag{12}$$

  which for a 5 MeV alpha on gold gives $r_{\text{min}} = 5 \times 10^{-14}$ m.
- In fact, an absolute measurement of $N(\theta)$ was shown to be consistent with a radius of the nucleus which is almost a tenth of this value, which is why the alpha particle does not induce fission in gold.

The Rutherford atom I

- Rutherford model: electrons orbiting nucleus.
- Orbital frequency? Coulomb force provides centripetal force, or

$$m\frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \tag{13}$$

  from which we can calculate a velocity of $v = 1.6 \times 10^6$ m/sec and a kinetic energy of 1.2×10^{-18} Joules or 7 eV.
- Oscillation frequency is

$$f = \frac{v}{2\pi r} = \sqrt{\frac{1}{2(2\pi)^3\epsilon_0}\frac{e^2}{mr^3}}. \tag{14}$$

  If we use $r = 10^{-10}$ m to represent atomic dimensions, we obtain $f \simeq 3 \times 10^{15}$ Hz, which corresponds to UV light with a wavelength of about 100 nm.
- Therefore it is not unreasonable to expect that the Rutherford model of the atom will somehow provide a way of explaining atomic spectra.

The Rutherford atom II

- How long will the electron be able to orbit the atom? The acceleration can be found from

$$ma = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}. \tag{15}$$

- In classical electrodynamics, one can calculate the power radiated by an accelerated charge by the Larmor formula of

$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{e^2 a^2}{c^3}, \tag{16}$$

  from which we find that the power radiated by the electron would be expected to be $P \simeq 4 \times 10^{-9}$ J/sec.
- The electron of energy 1.2×10^{-18} J would therefore be expected to last only 0.3 nanoseconds before it would have radiated all its energy and crashed into the nucleus!