Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Review from last lecture

- Relativistic momentum: $p_{1,y} = \gamma m_0 v_{2,y}$
- Perpendicular (to frame shift $\vec{v}$) forces ($\vec{F} \perp \vec{v}$): $\vec{F} = \gamma m_0 \vec{a}$.
- Parallel forces ($\vec{F} \parallel \vec{v}$): $\vec{F} = \gamma^3 m_0 \vec{a}$.
- Kinetic energy: $E_k = (\gamma - 1) m_0 c^2$.
- Total energy: $E = E_0 + E_k$ with $E_0 = mc^2$. Concept of energy equivalence of mass! Handy atomic energy unit: electron volt (1 eV=1.602×10^{-19} Joules).
- Conservation of momentum becomes $E^2 = E_0^2 + p^2 c^2$.

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Momentum transforms

We went from $(ct_1)^2 - x_1^2 - y_1^2 - z_1^2 = (ct_2)^2 - x_2^2 - y_2^2 - z_2^2$ to find transformations for position and time:

$$\begin{aligned} x_2 &= \gamma\left(x_1 - vt_1\right) \qquad \text{and} \qquad y_2 = y_1 \qquad \text{and} \qquad z_2 = z_1 \\ t_2 &= \gamma\left(t_1 - \frac{\beta}{c}x_1\right) \end{aligned}$$

From Eq. **??** of $(E_1/c)^2 - p_{x,1}^2 - p_{y,1}^2 - p_{z,1}^2 = (E_2/c)^2 - p_{x,2}^2 - p_{y,2}^2 - p_{z,2}^2$ we can find an equivalent Lorentz transform for momentum and energy:

$$\begin{aligned} p_{x,2} &= \gamma\left(p_{x,1} - v\left(\frac{E_1}{c^2}\right)\right) \qquad \text{and} \qquad p_{y,2} = p_{y,1}\text{and} \qquad p_{z,2} = p_{z,1} \qquad (1) \\ E_2 &= \gamma(E_1 - vp_{x,1}). \end{aligned}$$

This is somewhat startling, for it tells us that we need to worry about the Lorentz transformation in considering conservation of energy! We "pay energy" for the frame shift. . .

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Velocity from momentum and energy

Again by comparing $(ct_1)^2 - x_1^2 - y_1^2 - z_1^2 = (ct_2)^2 - x_2^2 - y_2^2 - z_2^2$ with $(E_1/c)^2 - p_{x,1}^2 - p_{y,1}^2 - p_{z,1}^2 = (E_2/c)^2 - p_{x,2}^2 - p_{y,2}^2 - p_{z,2}^2$ we can also find a particle's velocity v in terms of its total energy E and momentum p using

$$\vec{v} = \frac{\vec{p}}{\gamma m_0} = \frac{\vec{p}}{E/c^2} = \frac{\vec{p}c^2}{E}. \tag{2}$$

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Relativisic collision I

A particle A with rest mass m_0 and velocity $v_A = 0.80c$ in the $\hat{x}$ direction collides with an initially-stationary particle B with rest mass $2m_0$. Note that for $\beta = 4/5$ we can find $\gamma = \frac{5}{3}$. In the frame S_1, we then use $p_{1,y} = \gamma m_0 v_{2,y}$ and $E = \gamma m_0 c^2$ to find

$$\begin{aligned} p_{x,A,1} &= \gamma m_0 v_{x,A} = \frac{5}{3} m_0 \frac{4}{5} c = \frac{4}{3} m_0 c. \\ p_{y,A,1} &= p_{z,A,1} = 0 \\ E_{A,1} &= \gamma m_0 c^2 = \frac{5}{3} m_0 c^2 \end{aligned}$$

for particle A, and

$$\begin{aligned} p_{x,B,1} &= \gamma m_0 v_{x,B} = 0 \\ p_{y,B,1} &= p_{z,B,1} = 0 \\ E_{B,1} &= \gamma m_0 c^2 = 1(2m_0)c^2 \end{aligned}$$

for particle B.

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Relativistic collision II

The total energy in frame S_1 is then

$$E_1 = E_{A,1} + E_{B,1} = \left(\frac{5}{3} + 2\right) m_0 c^2 = \frac{11}{3} m_0 c^2,$$

or $E_1 = 3.67 m_0 c^2$.
Chose center-of-momentum frame. Using $p_{x,2} = \gamma\left(p_{x,1} - v(E/c^2)\right)$, we find

$$\begin{aligned} p_{x,A,2} + p_{x,B,2} &= 0 = \gamma\left[\left(p_{x,A,1} - v(E_{A,1}/c^2)\right) + \left(p_{x,B,1} - v(E_{A,1}/c^2)\right)\right] \\ &= \frac{\left(\frac{4}{3} m_0 c - \frac{5}{3} m_0 v\right) + \left(0 - 2 m_0 v\right)}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

The result of the final line will be zero if the numerator on the final line is zero, or $\left(\frac{4}{3}c - \frac{5}{3}v - 2v\right) m_0 = 0$ from which we obtain a velocity v of S_2 relative to S_1 of $v = (4/11)c$.

Relativistic collision III

The total energies of the individual particles in S_2 can be found using $E_2 = \gamma(E_1 v p_{x,1})$ to be

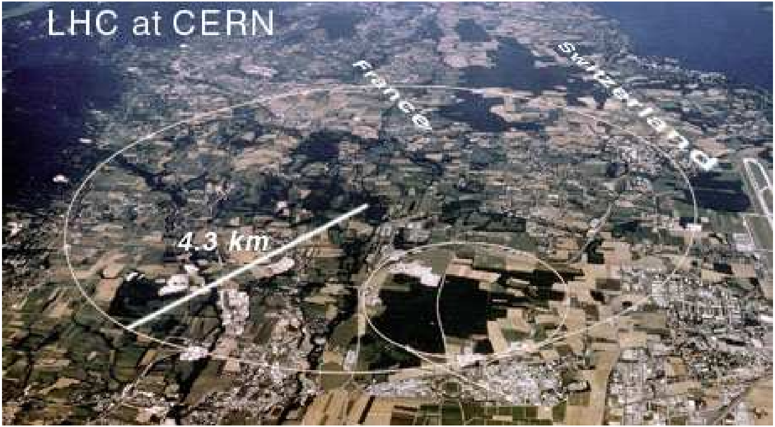
$$
\begin{aligned}
E_{A,2} &= \frac{E_{A,1} - v p_{x,A,1}}{\sqrt{1 - v^2/c^2}} \\
&= \frac{\frac{5}{3} m_0 c^2 - \left(\frac{4}{11} c\right)\left(\frac{4}{3} m_0 c\right)}{\sqrt{1 - \left(\frac{4}{11}\right)^2}} \\
&= 1.27 m_0 c^2 \\
E_{B,2} &= \frac{E_{B,1} - v p_{x,B,1}}{\sqrt{1 - v^2/c^2}} \\
&= \frac{2 m_0 c^2 - 0}{\sqrt{1 - \left(\frac{4}{11}\right)^2}} \\
&= 2.15 m_0 c^2,
\end{aligned}
$$

or $E_2 = 3.24 m_0 c^2$. Thus total energy available in the center-of-mass frame is less than the total energy in the fixed-target frame.
To increase total energy in a collision, use center-of-mass frame when relativistic effects are considered.

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Large Hadronic Collider or LHC

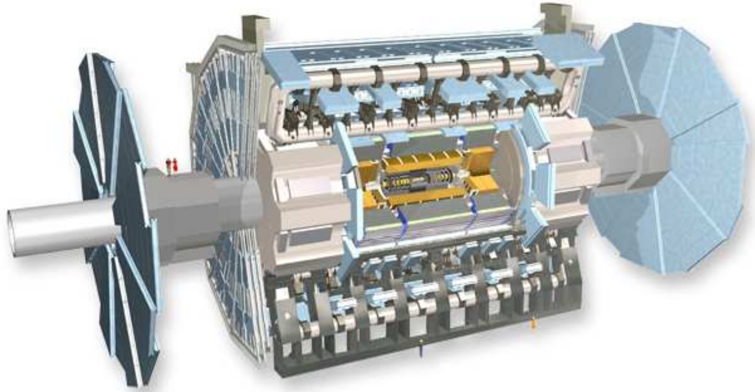
The next (last?) large accelerator for subatomic particle physics is nearing completion at CERN (acronym originally stood for Conseil Européen pour la Recherche Nucléaire) in Geneva. 7 TeV (7×10^{12} eV) protons against 7 TeV anti-protons.



Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

ATLAS at the LHC

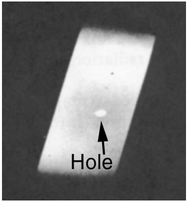
Several Stony Brook faculty are involved in experiments that will use this detector for capturing proton—anti-proton collisions at LHC. Notice the size of the people in this computer rendering?



Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Blackbody radiation

- Prosaic beginning to our story: how hot is a white-hot oven? Thomas Wedgwood (Wedgwood China still exists) notices in 1792 that all objects in the firing ovens shown the same shade of red no matter what their color is when cool.
- Classical thermodynamics has already uncovered Stefan's law which says that the total radiated power is $\propto T^4$. Note that heat transfer at room temperature is dominated by conduction and convection, not radiation. . .
- Heat a cavity to uniform temperature; observe radiation spectrum through a hole so small that we can neglect its cooling effects.
- How can we predict the spectrum?



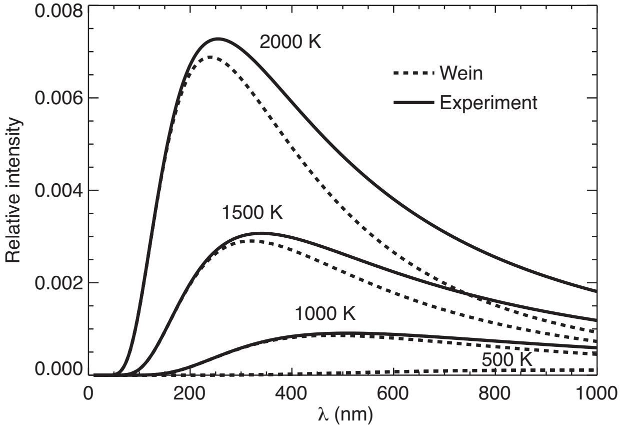
Measuring a black body

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

What does the spectrum look like?

Empirical fit by Wien in 1896:

$$\rho(\nu, T) = c_1 \nu^3 \exp[-c_2 \nu / T] \tag{3}$$



Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Elements of the calculation

- Our calculation will push us into a first look at statistical mechanics. We will want to know of two things:
  - *Density of available states* $g(E)$, which in this case will deal with the number of possible configurations of photons.
  - *Probability of occupying available states* $f(E)$, which in this case will deal with how many photons are likely to be in each of various states.
- You already know that electrons in atoms reside in discrete orbitals with particular energies. When you heat an atom up (add energy), which excited states are the electrons likely to be in?
- A crude example:
  - *Density of states* $g(E)$ says we have A plates of raw liver, and B bowls of ice cream available.
  - *Occupancy of states* $f(E)$ tells us that if we put N hungry kids in the room, what the occupancy $f(A)$ of state A is (how many kids will be eating raw liver), and what the occupancy $f(B)$ is (how many kids will be eating ice cream). You can guess that if $N \leq B$ we will find $f(B) = N$ and $f(A) = 0 \ldots$

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

States in blackbody radiation

- Assume that blackbody is a cavity of dimension $L_x \times L_y \times L_z$.
- If cavity is a conductor, must have nodes of electromagnetic waves at cavity walls.
- Therefore there must be an integer number of half-wavelengths along each dimension, or $n_x(\lambda/2) = L_x$.
- Since $c = \lambda\nu$ we can rearrange to obtain

$$\nu_x = n_x \frac{c}{2L_x} \quad (4)$$

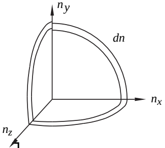
for allowed frequencies in the $\hat{x}$ direction. Since L_x can be continuously varied, so can ν_x, but n_x must be an integer.
- These are standing waves rather than traveling waves, so n_x is always positive.

Density of available states

- In three dimensions:

$$n = \sqrt{n_x^2 + n_y^2 + n_z^2}. \qquad (5)$$

- Density of allowed states for a range of modes n to $n + dn$: a shell in the positive octant.
- Volume of a sphere is $(4/3)\pi r^3$, so volume between n and $n + dn$ is



$$\begin{aligned}\frac{4}{3}\pi\left[(n+dn)^3 - n^3\right] &= \frac{4}{3}\pi n^3\left[\left(1+\frac{dn}{n}\right)^3 - 1\right] \\ &\simeq \frac{4}{3}\pi n^3\left[1 + 3\frac{dn}{n} - 1\right] \\ &= 4\pi n^2\, dn.\end{aligned}$$

Review
Momentum
Collisions
LHC
Blackbody
Cavity modes
Density of states
Thermodynamic occupancy
Blackbody redeaux
Blackbody measurements
Max Planck
Planck's fix
Interpreting Planck
Light
The electron
Photoelectric effect
Einstein and light
X rays

Density of states II

- We found that the shell of a sphere had $4\pi n^2\,dn$ available states. However, since we can only have positive n_x, n_y, and n_z, only one octant of sphere corresponds to physical states, so multiply by 1/8.
- Light can exist in two orthogonal polarizations, so multiply by 2.
- From Eq. 4 of $\nu_x = n_x(c/2L_x)$ we obtain

$$d(n_x) = d\left(\frac{2L_x}{c}\nu_x\right) \tag{6}$$

- Absolute density of available states is thus

$$\left(\frac{1}{8}\right)\cdot(2)\cdot(4\pi n^2)\,dn = \pi\left(\frac{2L}{c}\right)^3 \nu^2\,d\nu = V\,\frac{8\pi}{c^3}\nu^2\,d\nu. \tag{7}$$

- Volume-normalized result $\rho(\nu)$ (Planck, 1897) is then

$$\rho(\nu) = \frac{8\pi}{c^3}\nu^2\,d\nu. \tag{8}$$