PHY 251 Spring 2008: homework problem set 2, due Thursday, Feb. 14.

Serway problem 1.24

Answer: As seen from earth in frame S_1, we have $v_{1,x,A} = 0$ and $v_{1,y,A} = -0.90c$. If we shift into the frame of ship B, we need to do a frame shift of $v = +0.90c$ (we always define the frame shift axis to be in the $\vec{x}$ direction, and in this case it agrees with the problem's definition of the $\vec{x}$ direction). With that frame shift velocity, we can also calculate the Lorentz factor γ:

$$\gamma = \frac{1}{1-\beta^2} = \frac{1}{\sqrt{1-(0.90)^2}} = 2.29.$$

We can now use our relativistic velocity transformations to go from velocities observed in frame S_1 to those observed in frame S_2:

$$\begin{aligned} v_{2,x} &= \frac{v_{1,x}-v}{1-vv_{1,x}/c^2} = \frac{0c-0.9c}{1-(0.9c)(0c)/c^2} = -0.9c \\ v_{2,y} &= \frac{v_{1,y}}{\gamma[1-vv_{1,x}/c2]} = \frac{-0.90c}{2.29[1-(0.9c)(0c)/c^2]} = -0.39c \end{aligned}$$

We then have a net speed of $v = \sqrt{v_x^2 + v_y^2} = (\sqrt{(-0.90)^2 + (-0.39)^2})c = 0.98c$.

Serway problem 1.26

Answer: Your mantra should be that distances contract and time slows down. We observe a length for each spaceship of l_0/γ. Now we know that the proper lengths differ by a factor of three, which we'll write as $l_1 = 3l_2$ and since we see the same length for both we know that spaceship 1 is traveling faster relative to us than spaceship 2 is. The lengths we observe are the same due to the two different velocities of the two spaceships, or

$$\begin{aligned} \frac{l_1}{\gamma_1} &= \frac{l_2}{\gamma_2} \\ \gamma_1 &= \gamma_2\frac{l_1}{l_2} = \frac{1}{\sqrt{1-(0.35)^2}}\frac{3l_2}{l_2} = \frac{3}{\sqrt{1-(0.35)^2}} = 3.20 \end{aligned}$$

from which we obtain

$$\begin{aligned} \gamma &= (1-\beta^2)^{-1/2} \\ \gamma^{-2} &= 1-\beta^2 \\ \beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/(3.20)^2} = 0.95 \end{aligned}$$

so the fast spaceship is going at a speed of $0.95c$.

Serway problem 1.33

Answer: At the moment when the spacecraft is halfway between earth and Tau Ceti, it is at both the halfway point in the 12 lightyear journey that we would measure from earth, and the halfway point in the length-contracted distance on the spaceship. [With $\beta = 4/5$, we have $\gamma = 5/3$ as shown above so the total distance seen by the spaceship is $12/(5/3) = 7.2$ lightyears.] The point of the problem is that if one sees simultaneous flashes of light from both stars simultaneously while at the halfway point, then both stars exploded simultaneously because there was an equal light transit time to the halfway point. The speed of light is the same in all reference frames...

Serway problem 1.34

Answer: As with problem 1.19, we'll say that our frame on earth is frame S_1, and the frame traveling with the right hand rocket is frame S_2. We'll denote the left hand rocket as rocket A, and the right hand rocket as rocket B.

a. To find the proper lengths of the two rockets in their own frames, we need to determine the two Lorentz factors:

$$\begin{aligned}\gamma_A &= \frac{1}{\sqrt{1-(4/5)^2}} = \frac{1}{\sqrt{1-16/25}} = \frac{1}{\sqrt{9/25}} = \frac{5}{3} \\ \gamma_B &= \frac{1}{\sqrt{1-(3/5)^2}} = \frac{1}{\sqrt{1-9/25}} = \frac{1}{\sqrt{16/25}} = \frac{5}{4}.\end{aligned}$$

We see a contracted distance relative to what those on each rocket see, so rocket A has a length in its own frame of $50 \cdot (5/3) = 250/3$ meters, and rocket B has a length in its own frame of $50 \cdot (5/4) = 250/4$ meters.

b. To look at rocket A from rocket B's point of view, we must move frame S_1 to the left by $v = -0.6c$ in which case the speed of rocket A becomes

$$v_{2,A} = \frac{v_{1,A} - v}{1 - vv_{1,A}/c^2} = \frac{0.8c - (-0.6c)}{1 - (-0.6c)(0.8c)/c^2} = \frac{1.4c}{1.48} = 0.946c$$

The corresponding Lorentz factor is $\gamma = 1/\sqrt{1-(.946)^2} = 3.08$. The length of rocket A as viewed by rocket B is rocket A's proper length of 250/3 meters contracted by γ, or $(250/3)/3.08 = 27.0$ meters. Similarly, for rocket B as viewed by rocket A we first have to work out the velocity in a different frame S_2 which is now moving by $v = +0.8c$ relative to earth's frame S_1:

$$v_{2,B} = \frac{v_{1,B} - v}{1 - vv_{1,B}/c^2} = \frac{(-0.6c) - (0.8c)}{1 - (0.8c)(-0.6c)} = 0.946c$$

which, not surprisingly, gives us the same approach velocity and therefore the same Lorentz factor. As a result, the length of rocket B as seen by rocket A is $(250/4)/3.08 = 20.3$ meters.

c. To us on earth the two rockets are closing towards each other with a relative velocity of $1.4c$ so they will collide in a time of

$$t = \frac{x}{v} = \frac{2.52 \times 10^{12}}{1.4 \cdot 2.99 \times 10^{8}} = 6020 \text{ seconds}$$

or 1.67 hours.

d. From the point of view of rocket A, the earth-measured separation distance is reduced by a factor of γ_A, and rocket B approaches by a speed of $0.946c$, so the time before collision is

$$t = \frac{x}{v} = \frac{2.52 \times 10^{12}/(5/3)}{0.946 \cdot 2.99 \times 10^8} = 5350 \text{ seconds}$$

or 1.49 hours.

e. From the point of view of rocket B, the earth-measured separation distance is reduced by a factor of γ_B, and rocket A approaches it by a speed of $0.946c$, so the time before collision is

$$t = \frac{x}{v} = \frac{2.52 \times 10^{12}/(5/4)}{0.946 \cdot 2.99 \times 10^8} = 7130 \text{ seconds}$$

or 1.98 hours.

f. From the point of view of the crew on rocket A, 90 minutes or 1.50 hours in their own frame of reference is not quite enough time to evacuate the ship before collision.

Serway problem 1.36

Answer:

A) We could just do this first part by saying time dilates, but let's set it up in more detail to make part B easier. Let's say that Suzanne is in frame S_1, and Mark is in the frame S_2 that is moving at a speed v relative to Suzanne. Suzanne sees two light pulses from the same location; we'll set the origin of frame S_1 to be there so that we have $x_{1,A} = x_{1,B} = 0$. Susan sees the events separated in time according to $t_{1,B} = (t_{1,A} + \tau_1)$ where $\tau_1 = 3.00\ \mu$s. Mark, on the other hand, sees $t_{2,B} = (t_{1,A} + \tau_2)$ where $\tau_2 = 9.00\ \mu$s. Using the Lorentz transformations for time, we can write the time difference $t_{2,B} - t_{2,A}$ in Mark's frame as

$$\begin{aligned} t_{2,B} - t_{2,A} = \tau_2 &= \gamma(t_{1,B} - \frac{\beta}{c}x_{1,B}) - \gamma(t_{1,A} - \frac{\beta}{c}x_{1,A}) \\ &= \gamma\left[(t_{1,A} + \tau_1 - \frac{\beta}{c}0) - (t_{1,A} - \frac{\beta}{c}0)\right] \\ \tau_2 &= \gamma\tau_1. \end{aligned}$$

From this we find that $\gamma = \tau_2/\tau_1 = 9.00/3.00 = 3.00$, and and we can then use this to solve for β:

$$\begin{aligned} \gamma^2 &= \frac{1}{1-\beta^2} \\ 1-\beta^2 &= 1/\gamma^2 \\ \beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/9} = 0.94 \end{aligned}$$

as the velocity of Mark as seen by Suzanne.

B) To find out the distance separation in Mark's frame, we need to calculate $x_{2,A} - x_{2,B} = \chi_2$:

$$\chi_2 = x_{2,A} - x_{2,B} = \gamma(x_{1,A} - vt_{1,A}) - \gamma(x_{1,B} - vt_{1,B})$$

$$
\begin{aligned}
&= \gamma\left[(0 - vt_{1,A}) - (0 - v(t_{1,A} + \tau_1))\right] = \gamma\left[v\tau_1\right] \\
&= \gamma\beta c\tau_1 = 2.29\left[0.94 \cdot (3.00 \times 10^8 \text{ m/s}) \cdot (3.00 \times 10^{-6} \text{ s})\right] = 1940 \text{ meters}
\end{aligned}
$$

Serway problem 2.6

Answer: We have

$$\gamma m \frac{v^2}{R} = qvB \qquad \Rightarrow \qquad p = \gamma m v = qBR$$

If we're given B and R in mks units, we can calculate qBR in mks units for momentum (kg·m/sec). We then want to set qBR in mks units equal to some number p in units of MeV/c:

$$
\begin{aligned}
(1.602 \times 10^{-19}\text{ C})BR &= \left[p \text{ in } (\frac{\text{MeV}}{c})\right] \cdot \frac{10^6 \text{ eV}}{\text{MeV}} \cdot \frac{1.602 \times 10^{-19}\text{ J}}{\text{eV}} \cdot \frac{c}{3 \times 10^8 \text{ m/s}} \\
BR &= \left[p \text{ in } (\frac{\text{MeV}}{c})\right] \cdot \frac{10^6}{3 \times 10^8} = \left[p \text{ in } (\frac{\text{MeV}}{c})\right] \cdot \frac{1}{300} \\
300BR &= \left[p \text{ in } (\frac{\text{MeV}}{c})\right]
\end{aligned}
$$

Serway problem 2.8

Answer: The proton rest energy is $mc^2 = 938$ MeV. We have

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.95^2}} = 3.20$$

so the total energy is $\gamma mc^2 = 3000$ MeV and the kinetic energy is $(\gamma - 1)mc^2$=2060 MeV.

Serway problem 2.11

Answer: We have $E_k = (\gamma - 1)m_0c^2 = 50$ GeV, or

$$\gamma = 1 + \frac{E_k}{m_0c^2} = 1 + \frac{50,000 \text{ MeV}}{(938.3 \text{ MeV}/c^2)c^2} = 54.3.$$

Its speed can then be found from

$$\beta = (1 - 1/\gamma^2)^{1/2} \simeq 1 - \frac{1}{2\gamma^2} = 1 - \frac{1}{2 \cdot (54.3)^2} = 1 - 1.7 \times 10^{-4}$$

so it's traveling very close to the speed of light. It's much more accurate to report the difference from $\beta = 1$ than to say what the speed is in meters per second where a reader of your solution would have to look carefully at the difference five digits down…

Serway problem 2.14

Answer: Since $E = mc^2$ we can find the mass equivalent with

$$m = \frac{E}{c^2} \rightarrow \frac{4.0 \times 10^{26}\text{ J}}{s} \cdot \frac{1}{(3 \times 10^8)^2} = 4.4 \times 10^9 \text{ kg/s}$$

At this rate, the sun will shine for

$$\frac{2.0 \times 10^{30} \text{ kg}}{4.4 \times 10^{9} \text{ kg/s}} \cdot \frac{1 \text{ day}}{24 \cdot 60 \cdot 60 \text{ s}} \cdot \frac{1 \text{ year}}{365.25 \text{ days}} = 1.4 \times 10^{13} \text{ years}$$

Of course the rate of radiation would probably decline with mass loss so it would taper off rather than burn constantly to the end. . . But stars collapse and explode before that happens.

Serway problem 2.15

Answer: The kinetic energy of an electron with $m = 511 \times 10^3$ eV/c^2 is qV so we have

$$\begin{aligned} qV &= (\gamma - 1)mc^2 \\ \frac{qV}{mc^2} &= \gamma - 1 \\ \gamma &= 1 + \frac{qV}{mc^2} = 1 + \frac{5 \times 10^4 \text{ eV}}{511 \times 10^3 \text{ eV}} = 1.0978. \end{aligned}$$

The speed can then be found from

$$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/1.0978^2} = 0.412.$$

If we do this classically we find

$$\begin{aligned} qV &= \frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2 \\ \beta &= \sqrt{\frac{2qV}{mc^2}} = \sqrt{\frac{2 \cdot 5 \times 10^4}{511 \times 10^3}} = 0.442 \end{aligned}$$

The difference is then about 6.8%. This will probably affect the size of the electron beam on the TV screen because the electron focusing optics will not be quite set right. A bigger scanned electron spot on the phosphor will make the image appear fuzzier (pixels will blur into each other). It will also affect the brightness of the image because the amount of light given out by the phosphor is proportional to the energy of the electrons slamming into it. It will *not* affect the color of the image because that's determined by the dopant atoms in the phosphor; some phosphors are designed to glow red when excited, others blue, others green. . .

Serway problem 2.18

Answer: The mass of an electron in atomic mass units is

$$511 \times 10^3 \text{ eV}/c^2 \cdot \frac{1 \text{ u}}{931.494 \times 10^6 \text{ eV}/c^2} = 0.000556 \text{ u}$$

The mass difference Δm is then given by

$$54.9279 \text{ u} = (54.9244 \text{ u}) + (0.000556 \text{ u}) + \Delta m$$

giving $\Delta m = 0.0028$ u which we can express in MeV as

$$(0.0028\,\text{u}) \cdot \frac{931.494\,\text{MeV}/c^2}{1\,\text{u}} = 2.608\,\text{MeV}/c^2$$

We then go from an atom at rest to an atom and electron flying away from each other. Because the atom is massive and the electron is very light, the electron will carry nearly all of the kinetic energy so its maximum energy will be about 2.608 MeV/c^2.