PHY 251 Spring 2008: homework problem set 11, due Thursday, May 8.

Serway problem 14.4

Answer: The reaction ${}^{27}_{13}\mathrm{Al}(\alpha, n){}^{30}_{15}\mathrm{P}$ can also be written as ${}^{27}_{13}\mathrm{Al} +{}^{4}_{2}\alpha \rightarrow{}^{30}_{15}\mathrm{P} +{}^{1}_{0}n$ so that the Q of the reaction is

$$\frac{931.5\ \mathrm{MeV}}{\mathrm{amu}} \cdot (26.981\,538 + 4.002\,603 - 29.978\,314 - 1.008\,665) = -2.64\ \mathrm{MeV}$$

so the incoming alpha particle must have 2.64 MeV of energy after overcoming the Coulomb repulsion barrier to get close enough to be captured by the nuclear potential.

Serway problem 14.7

Answer: The value of Q for ${}^{4}_{2}\mathrm{He} +{}^{14}_{7}\mathrm{N} \rightarrow{}^{17}_{8}\mathrm{O} +{}^{1}_{1}\mathrm{H}$ is

$$Q = (4.002\,603 + 14.003\,074 - 16.999\,131 - 1.007\,825)\,\frac{931.494\ \mathrm{MeV}/c^2}{\mathrm{amu}}\,c^2 = 1.19\ \mathrm{MeV}.$$

Therefore we need the incoming α particle to have a kinetic energy of at least 1.19 MeV to drive the reaction. Next, we want to find the Q of ${}^{1}_{1}\mathrm{H} +{}^{7}_{3}\mathrm{Li} \rightarrow{}^{4}_{2}\mathrm{He} +{}^{4}_{2}\mathrm{He}$, which is

$$Q = (1.007\,825 + 7.016\,004 - 4.002\,603 - 4.002\,603)\,\frac{931.494\ \mathrm{MeV}/c^2}{\mathrm{amu}}\,c^2 = 17.35\ \mathrm{MeV}$$

so the alpha particles that emerge are very energetic.

Serway problem 14.22

Answer: In example 14.4 on page 513 of Serway, we find that one ${}^{235}\mathrm{U}$ fission event releases 208 MeV. Therefore the number x of fission events needed per day to generate 1000 MW is

$$x = (\frac{1\ \mathrm{decay}}{208 \times 10^6\ \mathrm{eV}}) \cdot (\frac{\mathrm{eV}}{1.602 \times 10^{-19}\ \mathrm{Joules}}) \cdot (\frac{10^9\ \mathrm{Joules}}{\mathrm{sec}}) \cdot (\frac{24 \cdot 60 \cdot 60\ \mathrm{sec}}{1\ \mathrm{day}})$$

or $x = 2.59 \times 10^{24}$ decays per day. The mass m of the number of ${}^{235}\mathrm{U}$ nuclei which must undergo fission in a day is then

$$m = (2.59 \times 10^{24}\ \mathrm{atoms}) \cdot (235.043 \frac{\mathrm{amu}}{\mathrm{atom}}) \cdot (1.661 \times 10^{-27}\ \frac{\mathrm{kg}}{\mathrm{amu}}) = 1.01\ \mathrm{kg}.$$

Because density ρ is mass per volume, we have

$$\begin{aligned} \rho &= \frac{m}{(4/3)\pi r^3} \\ r &= \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3\,(1010\ \mathrm{g})}{4\pi\,(19.0\ \mathrm{g/cm^3})}\right)^{1/3} = 2.33\ \mathrm{cm} \end{aligned}$$

or a sphere with a diameter of less than two inches.

Serway problem 14.24

Answer: The estimate for the total mass m of recoverable ^{235}U is

$$m = (10^9 \text{ tons U}) \cdot (\frac{2000 \text{ pounds}}{\text{ton}}) \cdot (\frac{1 \text{ kg}}{2.204 \text{ pounds}}) \cdot (\frac{0.007\ {}^{235}\text{U}}{\text{U}}) = 6.35 \times 10^9 \text{ kg}$$

The total amount of energy E that can be released from this is

$$E = (6.35{\times}10^9 \text{ kg}){\cdot}(\frac{1 \text{ amu}}{1.661 \times 10^{-27} \text{ kg}}){\cdot}(\frac{1 \text{ atom}}{235.043 \text{ amu}}){\cdot}(\frac{208 \times 10^6 \text{ eV}}{\text{atom}}){\cdot}(\frac{1.602 \times 10^{-19} \text{ Joule}}{\text{eV}})$$

or $E = 5.4 \times 10^{23}$ Joules. We therefore arrive at a time t over one could in principle deliver a power of 7×10^{12} Joules/sec:

$$t = \frac{5.4 \times 10^{23} \text{ J}}{7 \times 10^{12} \text{ J/sec}} \cdot \frac{1 \text{ year}}{365 \cdot 24 \cdot 3600 \text{ sec}} = 2500 \text{ years}$$

What energy crisis?

Serway problem 14.28

Answer: The number of tritium atoms is

$$(50 \text{ m}^3) \cdot (1.5 \times 10^{14} \frac{\text{atoms}}{\text{cm}^3}) \cdot (\frac{10^2 \text{ cm}}{\text{m}})^3 = 7.5 \times 10^{21} \text{ atoms.}$$

The activity is given by $\mathcal{A} = \lambda N$ or

$$\mathcal{A} = \lambda N = \frac{\ln(2)}{t_{1/2}} N = \frac{\ln(2)}{12 \text{ years}}{\cdot}(\frac{1 \text{ year}}{365 \cdot 24 \cdot 3600 \text{ sec}}){\cdot}(7.5{\times}10^{21} \text{ atoms}){\cdot}(\frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ decays/sec}})$$

or $\mathcal{A} = 370$ Ci. This is about a factor of 10^{-8} the radioactivity involved in fueling a fission plant.

Serway problem 14.30

Answer: For the reaction $4({}^1_1\text{H}) \rightarrow {}^4_2\text{ He} + 2e^+ + 2\nu + \gamma$, the energy released or Q can be found from the mass change:

$$Q = (4 \cdot 1.007\,825 - 4.002\,603 - 2 \cdot 0.000\,549) \cdot \frac{931.494 \text{ MeV/c}^2}{\text{amu}} = 25.7 \text{ MeV.}$$

The number x of these reactions that must occur per second is

$$x = (4 \times 10^{26} \frac{\text{J}}{\text{s}}) \cdot (\frac{1 \text{ reaction}}{25.7 \times 10^6 \text{ eV}}) \cdot (\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}) = 9.7 \times 10^{37}$$

or, since four protons go into one reaction, about 4×10^{38} protons per second. The mass per second converted into energy can be found from

$$m = \frac{E}{c^2} = \frac{4 \times 10^{26} \text{ J/s}}{(2.99 \times 10^9 \text{ m/s})^2} = 4.5 \times 10^7 \text{ kg/s.}$$

Serway problem 14.35

Answer: The total number N of ^{6}Li atoms available is

$$N = (2 \times 10^{10}\ \text{kg}) \cdot (0.075\ \frac{^6\text{Li}}{\text{Li}}) \cdot (\frac{1\ \text{atom}}{6\ \text{amu}}) \cdot (\frac{1\ \text{amu}}{1.661 \times 10^{-27}\ \text{kg}}) = 1.5 \times 10^{35}\ \text{atoms.}$$

The total energy E that can then be released is

$$E = (1.5 \times 10^{35}\ \text{atoms}) \cdot (\frac{22 \times 10^{6}\ \text{eV}}{\text{atom}}) \cdot (\frac{1.602 \times 10^{-19}\ \text{J}}{\text{eV}}) = 5.3 \times 10^{23}\ \text{Joules}$$

or about double the amount of energy in the total fossil fuel estimate.