PHY 251 Spring 2008: homework problem set 10, due Thursday, May 1.

Serway problem 13.1

Answer: Because of constant nuclear density we have $r = r_0 A^{1/3}$ with $r_0 = 1.2 \times 10^{-15}$ m so

$$\begin{aligned} {}^4_2\text{H has } r &= (1.2 \times 10^{-15})(4)^{1/3} = 1.9 \times 10^{-15} \text{ m} \\ {}^{238}_{92}\text{U has } r &= (1.2 \times 10^{-15})(238)^{1/3} = 7.4 \times 10^{-15} \text{ m} \end{aligned}$$

so the ratio is $(238/4)^{1/3} = 3.9$ which is modest for such a large mass ratio of $238/4 \simeq 60$.

Serway problem 13.5

Answer: The turn-around point is when all of the alpha particle's kinetic energy goes into electrostatic potential energy, or

$$\begin{aligned} E_k &= \frac{1}{4\pi\epsilon_0}\frac{Z_\alpha Z}{r} \\ \text{so } r &= \frac{Z_\alpha Z}{4\pi\epsilon_0 E_k} = \frac{(2)(79)}{4\pi(8.85 \times 10^{-12}) \cdot (0.5 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV})} = 4.5 \times 10^{-13} \text{ m} \end{aligned}$$

or 450 fm. To approach within 300 fm we must have a kinetic energy of

$$E_k = \frac{1}{4\pi\epsilon_0}\frac{(2)(79)(1.602 \times 10^{-19} \text{ Coulomb})^2}{300 \times 10^{-15}} = 1.21 \times 10^{-13} \text{ Joules}$$

or if we divide by 1.602×10^{-19} J/eV an energy of 0.75 MeV. This energy is small compared to the alpha particle rest mass of about $4 \cdot 940$ MeV so we can find the velocity using a non-relativistic formula:

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2E_k}{mc^2}} \cdot c = \sqrt{\frac{2 \cdot 0.75}{4 \cdot 940}} \cdot 2.99 \times 10^8 = 6.0 \times 10^6 \text{ m/s.}$$

Serway problem 13.9

Answer: We assume that the net atomic weight of copper m is due to a fraction x times the weight m_{63} of ^{63}Cu and a fraction $(1-x)$ of the weight m_{65} of ^{65}Cu. That is,

$$\begin{aligned} m &= x m_{63} + (1-x) m_{65} \\ m - m_{65} &= x(m_{63} - m_{65}) \\ x &= \frac{m - m_{65}}{m_{63} - m_{65}} = \frac{63.55 - 64.95}{62.95 - 64.95} = 0.70 \end{aligned}$$

so we have 70% ^{63}Cu and 30% ^{65}Cu.

Serway problem 13.16

Answer: The desired reaction is ${}^{43}_{20}\text{Ca}_{23} \rightarrow {}^{42}_{20}\text{Ca}_{22} + {}^{1}_{0} n_1$. Using Appendix B, we can get an estimate of the binding energy per nucleon for calcium nucleii from knowing the atomic mass of ${}^{40}_{20}$Ca:

$$\frac{931.5\ \text{MeV/amu}}{40\ \text{nucleons}}\left[(39.962\,591\ \text{amu}) - (20\ n)\cdot(1.008\,665\ \frac{\text{amu}}{n}) - (20\ p)\cdot(1.007\,825\ \frac{\text{amu}}{p})\right]$$
$$= -8.6\ \text{MeV per nucleon}$$

so that removing 1 neutron will cost 8.6 MeV. We can also use the full isotope mass tables at
http://physics.nist.gov/PhysRefData/Compositions/index.html
http://www-nds.iaea.org/nudat2/index.jsp
to do an exact calculation rather than an estimate:

$$[42.958\,766 - (41.958\,618 + 1.008\,665)]\cdot(931.5\ \text{MeV/amu}) = -7.93\ \text{MeV}.$$

The fact that we get a negative number means that this is not going to happen spontaneously.

Serway problem 13.17

Answer: Looking at Serway Fig. 13.10, we see that at $A = 200$ the binding energy is about 7.4 MeV/nucleon, while at $A = 100$ it is more like 8.3 MeV/nucleon. Therefore the energy released is

$$(2\cdot 100\ \text{nucleons}\cdot 8.3\ \text{MeV/nucleon}) - (200\ \text{nucleons}\cdot 7.4\ \text{MeV/nucleon}) = 180\ \text{MeV}.$$

Serway problem 13.22

Answer: The relationship between half-life $t_{1/2}$ and decay constant λ is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{8.04\ \text{days}}\cdot\frac{1\ \text{day}}{24\cdot 60\cdot 60\ \text{sec}} = 9.98\times 10^{-7}\ \text{sec}^{-1}$$

To get an activity $\mathcal{A}$ of 0.5 μCi, we must have

$$N = \frac{\mathcal{A}}{\lambda} = \frac{(0.5\times 10^{-6}\ \text{Ci})\cdot(3.7\times 10^{10}\ \text{decays/sec/Ci})}{9.98\times 10^{-7}} = 1.85\times 10^{10}\ \text{nuclei}.$$

Serway problem 13.26

Answer: The relationship between activity and number of nuclei is $\mathcal{A} = \lambda N$ giving

$$N = \frac{\mathcal{A}}{\lambda} = \frac{\mathcal{A}\,t_{1/2}}{\ln 2} = \frac{(0.2\times 10^{-6}\ \text{Ci})\cdot(3.7\times 10^{10}\ \frac{\text{decays/sec}}{\text{Ci}})\cdot(8.1\ \text{days})\cdot(24\cdot 3600\ \frac{\text{sec}}{\text{day}})}{\ln 2}$$
$$= 7.5\times 10^{9}\ \text{nuclei}$$

Serway problem 13.30

Answer: The radioactive tracer lets one easily track the loss of engine block material into the oil, since it is relatively easy to determine how much radioactive iron there is in the oil by looking for nuclear decays at a specific energy. We started out with 20 μCi of ^{59}Fe in the engine, and ^{59}Fe has a decay rate of

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{45.1 \text{ days}} \cdot \frac{1 \text{ day}}{24 \cdot 3600 \text{ sec}} = 1.78 \times 10^{-7} \text{ sec}^{-1}.$$

After running the engine for a thousand hours, it is observed that the activity in the oil is 800 disintegrations per minute per liter of oil or a total rate of

$$\mathcal{A} = \frac{800 \text{ decays}}{\text{minute} \cdot \text{liter}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot (6.5 \text{ liters}) = 86.7 \text{ decays/sec}$$

or $(86.7/3.7 \times 10^{10}) = 2.34 \times 10^{-9}$ curies (Ci). If the sample were a thousand hours or $(1000 \cdot 3600) = 3.6 \times 10^{6}$ seconds fresher, one would have expected this amount of iron in the oil to have produced an activity calculated from $\mathcal{A} = \mathcal{A}_0 \exp[-\lambda t]$ to be

$$\mathcal{A}_0 = \mathcal{A} \exp[\lambda t] = (2.34 \times 10^{-9} \text{ Ci}) \exp\left[(1.78 \times 10^{-7}) \cdot (3.6 \times 10^{6})\right] = 4.44 \times 10^{-9} \text{ Ci}.$$

Therefore the fraction x of iron that was ground away and went into the oil is

$$x = \frac{4.44 \times 10^{-9} \text{ Ci}}{20 \times 10^{-6} \text{ Ci}} = 2.22 \times 10^{-4}$$

or a mass of $(2.22 \times 10^{-4}) \cdot (0.2 \text{ kg})$ or 4.44×10^{-5} kg. The mass worn per hour is a thousandth of this, or 4.4×10^{-8} kg/h.

Serway problem 13.33

Answer: The half-life of ^{14}C is 5730 years, so the decay rate is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730 \text{ years}} \cdot \frac{1 \text{ yr}}{365 \text{ d}} \cdot \frac{1 \text{ d}}{24 \cdot 3600 \text{ sec}} = 3.83 \times 10^{-12} \text{ sec}^{-1}.$$

In “fresh” carbon the ratio of ^{14}C/^{12}C is 1.3×10^{-12}, so one would expect 25 g of fresh carbon to have a number N_0 of ^{14}C atoms of

$$\frac{25 \text{ g}}{12.0 \text{ g/mol}} \cdot (6.02 \times 10^{23} \text{ atoms/mol}) \cdot (1.3 \times 10^{-12} \frac{^{14}\text{C}}{^{12}\text{C}}) = 1.6 \times 10^{12} \text{ atoms } ^{14}\text{C}.$$

The activity of fresh carbon would then be

$$\mathcal{A}_0 = \lambda N_0 = 3.83 \times 10^{-12} \cdot 1.6 \times 10^{12} = 6.1 \text{ decays/sec}$$

or 366 decays/minute. If 25,000 years or

$$(2.5 \times 10^{4} \text{ yr}) \cdot (365 \frac{\text{day}}{\text{yr}}) \cdot (24 \cdot 3600 \frac{\text{sec}}{\text{day}}) = 7.89 \times 10^{11} \text{ seconds}$$

have gone by, we would expect the activity to now be

$$\begin{aligned}\mathcal{A} &= \mathcal{A}_0 \exp[-\lambda t] = (366 \text{ decays/min}) \exp\left[-(3.83 \times 10^{-12} \text{ sec}^{-1}) \cdot (7.89 \times 10^{11} \text{ sec})\right] \\ &= 18 \text{ decays/min}.\end{aligned}$$

If the background signal level were only 20 counts/minute and we had 100% efficiency, the signal would be about the same as the background so that we would have to be very careful about background subtraction, knowing the detector efficiency, counting for a long time, and so on.

Serway problem 13.34

Answer: First of all, the decay rate of ^{90}Sr is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{27.7 \text{ yr}} \cdot \frac{1 \text{ yr}}{365 \text{ dy}} \cdot \frac{1 \text{ dy}}{24 \cdot 3600 \text{ sec}} = 7.93 \times 10^{-10} \text{ sec}^{-1}.$$

At the time of the accident, the activity is

$$\mathcal{A}_0 = \frac{5 \times 10^6 \text{ Ci}}{10^4 \text{ km}^2} \cdot (\frac{1 \text{ km}}{10^3 \text{ m}})^2 \cdot (\frac{10^6 \ \mu\text{Ci}}{\text{Ci}}) = 500 \ \mu\text{Ci/m}^2.$$

To get this to die down to $\mathcal{A} = 2 \ \mu$Ci/m^2, we manipulate $\mathcal{A} = \mathcal{A}_0 \exp[-\lambda t]$ to find

$$t = -\frac{\ln(\mathcal{A}/\mathcal{A}_0)}{\lambda} = -\frac{\ln(2/500)}{7.93 \times 10^{-10} \text{ sec}^{-1}} = 7.0 \times 10^9 \text{ sec}$$

or

$$(7.0 \times 10^9 \text{ sec}) \cdot \frac{1 \text{ day}}{24 \cdot 3600 \text{ sec}} \cdot \frac{1 \text{ year}}{365 \text{ days}} = 220 \text{ years}$$

so a catastrophic reactor accident can make an area unusable for quite a long time.

Serway problem 13.41

Answer: The energy released in $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}$ is

$$(931.5 \ \frac{\text{MeV}}{\text{amu}}) \cdot (238.050\,785 - 234.043\,595 - 4.002\,603) = 4.27 \text{ MeV}.$$

Serway problem 13.44

Answer: If we presume that the daughter nucleus does not recoil, then the kinetic energy of the alpha particle is just

$$Q = (M_{Rn} - M_\alpha - M_{Po}) \times 931.494 = 6.39 \text{ MeV}.$$

Serway problem 13.45

Answer: We have three suggested nuclear decays:

a) ${}^{40}_{20}\mathrm{Ca}_{20} \rightarrow^{40}_{19} \mathrm{K}_{20} + \nu$: this involves electron capture (see page 489 of Serway), so we should really write it as ${}^{40}_{20}\mathrm{Ca}_{20} + e^- \rightarrow^{40}_{19} \mathrm{K}_{20} + \nu$. We then have to look at the mass change:
$39.962\,591 + 0.000\,549 \rightarrow 39.963\,999 + 0$
$39.963\,140 \rightarrow 39.963\,999$
Since this would involve an *increase* in mass, energy would have to be added so it's not going to happen spontaneously.

b) ${}^{98}_{44}\mathrm{Ru} \rightarrow^{4}_{2} \mathrm{He} +^{94}_{42} \mathrm{Mo}$ has a mass change of
$97.905\,288 \rightarrow 4.002\,603 + 93.905\,088$
$97.905\,288 \rightarrow 97.907\,691$
which again involves an *increase* in mass so it won't happen spontaneously.

c) ${}^{144}_{60}\mathrm{Nd} \rightarrow^{4}_{2} \mathrm{He} +^{140}_{58} \mathrm{Ce}$ has a mass change of
$143.910\,083 \rightarrow 4.002\,603 + 139.905\,434$
$143.910\,083 \rightarrow 143.908\,040$
which involves a mass *decrease* so energy can be released and it will happen spontaneously.