Physics 251 exam 3, May 14, 2008. You may use a calculator, and the equation sheet that I have provided. Here are some atomic masses (in atomic mass units 1u=931.494 MeV/c^2):
${}^4_2$He: 4.002603, ${}^{12}_6$C: 12.0000, ${}^{55}_{24}$Cr: 54.9279, ${}^{55}_{25}$Mn: 54.9244, ${}^{235}_{92}$U=235.043922, ${}^{238}_{92}$U=238.050784, ${}^{239}_{94}$Pu=239.052156

1. A smug city slicker bets a farmer that he can't get his 10 m long ladder into a 8 m long shed. The farmer, who reads Einstein each day after milking his cows, takes him up on the bet. He tells the city slicker to stand to the side of the shed and look in the windows at each end, and the farmer then runs fast as he can through the shed while carrying the ladder. How fast does the farmer have to run to win the bet? While on the run, how long does the shed appear to the farmer, and does the farmer ever think his ladder is entirely inside the shed? (3 answers required).
Answer: For the city slicker in the stationary frame S_1 to observe the ladder as being completely inside the shed while the farmer runs by in the farmer's frame S_2, we must have a Lorentz contraction calculated from $L_1 = (1/\gamma)L_2$. Therefore
$$\gamma = \frac{L_2}{L_1} = \frac{10\text{ m}}{8\text{ m}} = \frac{5}{4}.$$
Now $\gamma^2 = 1/(1-\beta^2)$ so
$$\beta = \sqrt{1-1/\gamma^2} = \sqrt{1-4^2/5^2} = \sqrt{9/25} = 3/5$$
or $v = 0.60c$. Now, to the farmer the shed appears to be moving toward him at $v = 0.60c$, so its length is Lorentz contracted according to $\gamma = 5/4$, giving an apparent shed length of $(8\text{ m})/(5/4) = 6.4$ m. As a result, the farmer never sees the ladder as being completely inside the shed! But the farmer knows what the city slicker sees, and collects on the bet...

2. ^{239}Pu alpha decays with a half-life of 2.41×10^4 years. Compute the power output, in watts, which could be obtained (at 100% efficiency) from 2.0 kg of "fresh" ^{239}Pu. Thermal power sources like this have been used to power some craft launched to deep space where sunlight is weak; a recent example is the Cassini probe to Saturn (the team was led by a woman who earned her B.A. in Astronomy at Stony Brook).
Answer: The reaction is ${}^{239}_{94}\text{Pu} \rightarrow {}^4_2\,\alpha + {}^{235}_{92}$ U. The energy released per decay is
$$Q = m_{\text{Pu}} - m_\alpha - m_{\text{U}} = 239.052156 - 4.002603 - 235.043922 = 0.00563049\text{ u}$$
or $0.00563 \cdot 931.494 = 5.29$ MeV. Now we will want to know the number of seconds per year:
$$(60\text{ s/m}) \cdot (60\text{ m/h}) \cdot (24\text{ h/d}) \cdot (365\text{ d/y}) = 3.15 \times 10^7\text{ s/y}.$$
The activity is given by $R = \lambda N$ or, since $t_{1/2} = \log 2/\lambda$, the activity is
$$\begin{aligned} R &= \lambda N = \frac{\log 2}{t_{1/2}} \cdot \frac{m\,N_A}{A} \\ &= \frac{\log 2}{2.41 \times 10^4\text{ y}} \cdot \frac{1\text{ y}}{3.15 \times 10^7\text{ s}} \cdot \frac{2 \times 10^3\text{ g}}{239.052\text{ g/mole}} \cdot \frac{6.02 \times 10^{23}\text{ atoms}}{\text{mole}} \\ &= 4.6 \times 10^{12}\,\frac{\text{decays}}{\text{sec}} \end{aligned}$$

The power is then

$$\frac{5.29\times10^6\ \text{eV}}{\text{decay}}\cdot 4.59\times10^{12}\frac{\text{decays}}{\text{sec}}\cdot 1.602\times10^{-19}\frac{\text{J}}{\text{eV}} = 3.9\ \text{W}$$

3. Calculate the shortest wavelength transition in hydrogen. A distant galaxy is observed to have this transition appear in our measurements at a wavelength of 250 nm. How fast is the galaxy receding from us?
Answer: The shortest wavelength transition in hydrogen comes from the highest energy difference which is $E_2 - E_1 = (-13.6/2^2) - (-13.6/1^2) = (3/4)13.6$ eV $= 10.2$ eV or a waelength of $\lambda = hc/\Delta E = 1240$ eV·nm/10.2 eV=121 nm or $\nu_0 = c/\lambda_0$. If this is shifted to $\nu = c/\lambda$ with $\lambda = 250$ nm we have a relativistic Doppler shift of (using $\theta = 0$ for an emitter moving away, or $\cos\theta = 1$)

$$\begin{aligned}
\nu &= \frac{\nu_0}{\gamma[1+(v/c)]} \\
\gamma(1+\beta) &= \frac{\nu_0}{\nu} = \frac{c/\lambda_0}{c/\lambda} = \frac{\lambda}{\lambda_0} \equiv x = \frac{250\ \text{nm}}{122\ \text{nm}} \\
\frac{1+\beta}{\sqrt{1-\beta^2}} &= x \\
\frac{(1+\beta)(1+\beta)}{(1-\beta)(1+\beta)} = \frac{1+\beta}{1-\beta} &= x^2 \\
1+\beta &= x^2 - \beta x^2 \\
\beta(x^2+1) &= x^2 - 1 \\
\beta &= \frac{x^2-1}{x^2+1} = \frac{(250/122)^2-1}{(250/122)^2+1} = 0.61
\end{aligned}$$

so the galaxy is receding at a speed of $v = 0.61c$.

4. The half-life of ^{131}I is 8.04 days. A) Calculate the decay constant λ for this isotope. B) Find the number of ^{131}I nuclei, and the mass of the source, necessary to produce a sample with an activity of 0.5 μCi.
Answer: The relationship between half-life $t_{1/2}$ and decay constant λ is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{8.04\ \text{days}}\cdot\frac{1\ \text{day}}{24\cdot60\cdot60\ \text{sec}} = 9.98\times10^{-7}\ \text{sec}^{-1}$$

To get an activity $\mathcal{A}$ of 0.5 μCi, we must have

$$N = \frac{\mathcal{A}}{\lambda} = \frac{(0.5\times10^{-6}\ \text{Ci})\cdot(3.7\times10^{10}\ \text{decays/sec/Ci})}{9.98\times10^{-7}} = 1.85\times10^{10}\ \text{nuclei.}$$

The mass of this source is

$$(1.85\times10^{10}\ \text{atoms})\cdot(\frac{\text{mole}}{6.02\times10^{23}\ \text{atoms}})\cdot(\frac{131\ \text{grams}}{\text{mole}}) = 4.03\times10^{-12}\ \text{grams.}$$

5. A ^{55}Cr nucleus undergoes beta decay. What is the maximum kinetic energy and the velocity of the emitted electron? (When the electron is at this maximum, the neutrino carries negligible kinetic energy and can thus be ignored).
Answer: The mass of an electron in atomic mass units is

$$510.998 \times 10^3 \text{ eV}/c^2 \cdot \frac{1 \text{ u}}{931.494 \times 10^6 \text{ eV}/c^2} = 0.0005486 \text{ u}$$

If $^{55}_{24}$Cr undergoes beta decay, we have (almost) no change in mass but we need to change a neutron to a proton to balance out the charge of the electron, so the decay is to $^{55}_{25}$Mn. The mass difference Δm in the decay $^{55}_{24}\text{Cr} \rightarrow ^{55}_{25}\text{Mn} + e^-$ is then given by

$$54.9279 \text{ u} = (54.9244 \text{ u}) + (0.00055 \text{ u}) + \Delta m$$

giving $\Delta m = 0.0030$ u which we can express in MeV as

$$(0.0030 \text{ u}) \cdot \frac{931.494 \text{ MeV}/c^2}{1 \text{ u}} = 2.79 \text{ MeV}/c^2$$

We then go from an atom at rest to an atom and electron flying away from each other. Because the atom is massive and the electron is very light, the electron will carry nearly all of the kinetic energy so its maximum energy will be about 2.79 MeV/c^2. Its velocity can be found from KE=$(\gamma - 1)mc^2$ giving

$$\gamma = 1 + \frac{\text{KE}}{mc^2} = 1 + \frac{2.79 \times 10^6 \text{ eV}}{510.998 \times 10^3 \text{ eV}} = 6.46$$

so the electron is quite relativistic. We can find the velocity from

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-\beta^2}} \\ \frac{1}{\gamma^2} &= 1 - \beta^2 \\ \beta &= \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/6.46^2} = 0.988 \end{aligned}$$

so $v = 0.988c$.

6. You know that the ratio of ^{14}C/^{12}C is 1.3×10^{-12} for "fresh" carbon such as is taken up in plants, animals, *etc.*, and that the half-life of ^{14}C is 5730 years. You are given a 100 g sample of "old" carbon from an archaeological site which has an activity of 400 decays/minute. How old is this sample?
Answer: The time constant for decay is

$$\lambda = \frac{\log(2)}{t_{1/2}} = \frac{0.693}{5730 \text{ y}} \frac{1 \text{ y}}{365 \text{ d}} \frac{1 \text{ d}}{24 \text{ h}} \frac{1 \text{ h}}{3600 \text{ s}} = 3.83 \times 10^{-12} \text{ s}^{-1},$$

The intial number of ^{12}C atoms is

$$\frac{100 \text{ g}}{12.0 \text{ g/mol}} \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = 5.02 \times 10^{24} \text{ atoms}$$

The initial number of ^{14}C atoms is $1.3 \times 10^{-12} \cdot 5.02 \times 10^{24} = 6.5 \times 10^{12}$ atoms. The activity of 100 g of "fresh" carbon due to its ^{14}C mass fraction is thus

$$R_0 = N_0 \lambda = (6.5 \times 10^{12} \text{ atoms}) \cdot (3.83 \times 10^{-12} \text{ s}^{-1}) = 25 \text{ decays/s}$$

or 1500 decays/minute. The activity of the sample has thus declined according to $R = R_0 e^{-\lambda t}$ so

$$\begin{aligned} R/R_0 &= e^{-\lambda t} \\ \log(R/R_0) &= -\lambda t \\ t &= -\log(R/R_0)/\lambda = -t_{1/2} \log(R/R_0)/\log(2) \\ &= -(5730 \text{ years}) \log(400/1500)/\log(2) = 10,900 \text{ years} \end{aligned}$$

7. Filling no more than two half-pages in a blue book, describe how it is that discrete states in individual atoms become energy bands in multi-atom solids; state what the Fermi-Dirac distribution has to say about state occupancy; and show the relationship between bands and the Fermi energy in conductors, insulators, and semiconductors.

8. Pick out what you think are the five most important ideas/advances of modern physics. Put them in proper chronological order. For each, give an approximate date and a discoverer's name, indicate the crucial experiment associated with the advance, and indicate how it led to the next of your five ideas. This is not an exercise in writing a long essay; your answer should fit on two or three blue book half-pages.