Physics 251 final exam, May 14, 2007. You may use a calculator, and the equation sheet that I have provided.

1. What's the de Brogile wavelength of an electron accelerated through a potential of 400,000 volts?
   Answer: The kinetic energy is $E_k = (\gamma - 1)m_e c^2$ so we have

$$\gamma = 1 + \frac{(400\text{ kV}) \cdot (1\text{ electron})}{(511\text{ keV}/c^2) \cdot c^2} = 1.78.$$

   The de Broglie wavelength is then

$$\lambda = \frac{h}{p} = \frac{hc}{\gamma m_e vc} = \frac{hc}{\gamma\beta m_e c^2}$$

   so we need β:

$$\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-\beta^2}} \\
\gamma^2 &= \frac{1}{1-\beta^2} \\
1-\beta^2 &= 1/\gamma^2 \\
\beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/(1.78^2)} = 0.827.
\end{aligned}$$

   We then have

$$\lambda = \frac{hc}{\gamma\beta m_e c^2} = \frac{1240\text{ eV}\cdot\text{nm}}{1.78 \cdot 0.827 \cdot (511 \times 10^3\text{ eV})} = 0.00165\text{ nm}.$$

2. Sodium is a monovalent metal having a density of 0.971 g/cm^3, and an atomic weight of 23.0 g/mol. Calculate a) the free electron density; b) the Fermi energy E_F; and c) the Fermi velocity v_F of the free electrons.
   Answer: Because sodium is monovalent we have 1 free electron per atom, and

$$\frac{N}{V} = n = \frac{1\ e}{\text{atom}} \cdot \frac{(0.971\text{ g/cm}^3) \cdot (6.02 \times 10^{23}\text{ atoms/mol})}{23.0\text{ g/mol}} = 2.54 \times 10^{22}\ e/\text{cm}^3.$$

   The Fermi energy can be found from

$$\begin{aligned}
E_F &= \frac{h^2}{2m_e}\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3} = \frac{(hc)^2}{2m_e c^2}\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3} \\
&= \frac{(1240\text{ eV}\cdot\text{nm})^2}{2 \cdot 511 \times 10^3} \cdot \left(\frac{3}{8\pi}(2.54 \times 10^{22}\text{ cm}^{-3}) \cdot (\frac{1\text{ cm}}{10^7\text{ nm}})\right)^{2/3} = 3.15\text{ eV}.
\end{aligned}$$

   Because $(1/2)mv_F^2 = E_F$ we have

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2E_F}{m_e c^2}}\,c = \sqrt{\frac{2 \cdot 3.15}{511 \times 10^3}}\,2.99 \times 10^8 = 1.05 \times 10^6\text{ m/sec}.$$

3. A freshly prepared sample of ^{64}Cu has an activity of 9.63 mCi. After an elapsed time of 4 hours, its activity is 7.74 mCi. Find a) the decay constant and b) the half-life of the isotope. c) What's the mass of the sample? d) What's its activity after a week has elapsed?
Answer: Activity decays as $\mathcal{A} = \mathcal{A}_0 e^{-\lambda t}$ giving

$$\lambda = -\frac{1}{t}\ln\left(\frac{\mathcal{A}}{\mathcal{A}_0}\right) = -\frac{1}{4\cdot 3600\text{ sec}}\ln(\frac{7.74}{9.63}) = 1.52\times 10^{-5}\text{ sec}^{-1}$$

and $t_{1/2} = \ln(2)/\lambda = 4.56\times 10^4$ seconds or 12.7 hours. The initial number of atoms N_0 can be found from $\mathcal{A}_0 = \lambda N_0$ giving

$$N_0 = \frac{\mathcal{A}_0}{\lambda} = \frac{(9.63\times 10^{-3}\text{ Ci})\cdot(3.7\times 10^{10}\text{ atoms/sec/Ci})}{1.52\times 10^{-5}\text{ sec}} = 2.34\times 10^{13}\text{ atoms}$$

and a mass m of

$$m = \frac{(2.34\times 10^{13}\text{ atoms})\cdot(64\text{ g/mol})}{6.02\times 10^{23}\text{ atoms/mol}} = 2.49\times 10^{-9}\text{ g.}$$

Finally, after a week has elapsed the activity is

$$\mathcal{A} = \mathcal{A}_0 e^{-\lambda t} = \mathcal{A}_0\exp[-\log(2)\frac{t}{t_{1/2}}] = (9.63\text{ mCi})\exp[-\log(2)\frac{24\cdot 7\text{ hours}}{12.7\text{ hours}}] = 0.00100\text{ mCi}$$

or 1.00 μCi.

4. With the knowledge that ^{235}U fission releases about 208 MeV per atom, and assuming that 70% of the ^{235}U fuel load is used up over a five year period, how many kg of ^{235}U is required to load a 1200 MW nuclear power plant?
Answer: To produce 1200 MW for five years, we need a total energy of

$$(1200\times 10^6\ \frac{\text{Joules}}{\text{sec}})\cdot(5\text{ yr})\cdot(\frac{365\text{ d}}{\text{yr}})\cdot(\frac{24\cdot 3600\text{ sec}}{\text{d}}) = 1.89\times 10^{17}\text{ Joules}$$

or, using 1.602×10^{-19} J/eV, a total energy of 1.18×10^{30} MeV. That means we need to have $(1.18\times 10^{30}\text{ MeV})/(208\text{ MeV/fission}) = 5.67\times 10^{27}$ fission events, or that same number of ^{235}U nuclei. The mass needed if we use 100% of the fuel load is

$$(5.67\times 10^{27}\text{ atoms})\cdot(\frac{235\text{ g/mol}}{6.02\times 10^{23}\text{ atoms/mol}})\cdot(\frac{1\text{ kg}}{10^3\text{ g}}) = 2210\text{ kg}$$

or about two tons. However, we are able to use only 70% of the ^{235}U up, so we need 1/0.70 more, or 3160 kg or about three tons. Not bad for five years of running, right?

5. The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.35c$, determine the speed of the faster spaceship.

Answer: We observe a length for each spaceship of l_0/γ. Now we know that the proper lengths differ by a factor of three, which we'll write as $l_1 = 3l_2$ and since we see the same length for both we know that spaceship 1 is traveling faster relative to us than spaceship 2 is. The lengths we observe are the same due to the two different velocities of the two spaceships, or

$$\begin{aligned}\frac{l_1}{\gamma_1} &= \frac{l_2}{\gamma_2}\\ \gamma_1 &= \gamma_2\frac{l_1}{l_2} = \frac{1}{\sqrt{1-(0.35)^2}}\frac{3l_2}{l_2} = \frac{3}{\sqrt{1-(0.35)^2}} = 3.20\end{aligned}$$

from which we obtain

$$\begin{aligned}\gamma &= (1-\beta^2)^{-1/2}\\ \gamma^{-2} &= 1-\beta^2\\ \beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/(3.20)^2} = 0.95\end{aligned}$$

so the fast spaceship is going at a speed of $0.95c$.

6. Construct an energy level diagram for the first four states of a Li^{2+} ion, and indicate the wavelengths for all $|\Delta n| = 1$ transitions.
   Answer: The energies are given by $E_n = -(13.6\text{ eV})Z^2/n^2$ with $Z = 3$ so for $n = 1, 2, 3, 4$ we have $E_{1,2,3,4} = [-122.4, -30.6, -13.6, -7.65]$ eV. The transition wavelengths are $\lambda = hc/\Delta E = (1240\text{ eV}\cdot\text{nm})/\Delta E$ or $1240/(122.4-30.6) = 13.5$ nm, $1240/(30.6-13.6) = 72.9$ nm, and $1240/(13.6-7.65) = 208$ nm.

7. Write in a two-page spread in your blue book a short discussion of the differences between insulators, conductors, and semiconductors. Include a discussion of bands, the Fermi energy and distribution, and dopants.

8. Write in a two-page spread in your blue book a short synopsis of the development of quantum mechanics, starting with Planck and ending with the Schrödinger equation. Include formulæ.

9. and 10. Pick two of the three topics below. Write a two-page spread in your blue book on one topic, and a two-page spread in your blue book on the other topic. Emphasize physics rather than engineering details.

   a. How does a fission reactor work?
   b. How does a fission-fusion-fission bomb work?
   c. How might a fusion power plant work?