Physics 251 exam 2, April 23, 2008. You may use a calculator, and the equation sheet that I have provided.

1. What's the de Broglie wavelength of an electron accelerated through a potential difference of 5,000 Volts?
   Answer: The electron has a kinetic energy of 5 keV, which is well below its rest mass of 511 keV/c^2 so we can do this problem with classical momentum. We then have $E_k = p^2/2m$ or $p = \sqrt{2mE_k}$ so the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2 E_k}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2 \cdot (511 \times 10^3 \text{ eV}) \cdot (5 \times 10^3 \text{ eV})}} = 0.017 \text{ nm}.$$

   Note that $E = hc/\lambda$ applies only to massless particles (photons)!

2. Write out the electronic configuration for neon ($Z = 10$). Write out the values for the set of quantum numbers n, ℓ, m_ℓ, and m_s for each of the electrons in neon.
   Answer: Neon has 10 electrons in the configuration $1s^2 2s^2 2p^6$ with quantum numbers as follows:

Electron #	n	ℓ	m_ℓ	m_s
1	1	0	0	+1/2
2	1	0	0	-1/2
3	2	0	0	+1/2
4	2	0	0	-1/2
5	2	1	+1	+1/2
6	2	1	0	+1/2
7	2	1	-1	+1/2
8	2	1	+1	-1/2
9	2	1	0	-1/2
10	2	1	-1	-1/2

3. Here are some radial wavefunctions for the Schrödinger equation for the hydrogen atom:

$$\begin{aligned} R_{10} &= \frac{2}{a_0^{3/2}} \exp[-r/a_0] \\ R_{20} &= \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) \exp[-r/2a_0] \\ R_{21} &= \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} \exp[-r/2a_0] \end{aligned}$$

   Calculate the average radius $r_{av} = \langle r \rangle$ for a $1s$ orbital, and compare it with the Bohr radius.
   Answer: This problem is like homework problem 7.17 and others where we used $\langle f \rangle =$

$\int f\,\psi^*\psi$. The radial part of the $1s$ wavefunction is given by R_{10}. We then want

$$\begin{aligned} r_{\text{av}} = \langle r \rangle &= \int_0^\infty r\, R_{10}(r)^*\, R_{10}(r)\, r^2\, dr \\ &= \int_0^\infty \frac{4}{a_0^3} r^3 \exp\left[-\frac{2}{a_0} r\right]\, dr = \frac{4}{a_0^3}\int_0^\infty r^n\, \exp[-ax]\, dx \text{ with } a = \frac{2}{a_0} \text{ and } n = 3 \\ &= \frac{4}{a_0^3}\,\frac{3!}{(2/a_0)^4} = \frac{4}{a_0^3}\,\frac{6}{16/a_0^4} = \frac{3}{2}\,a_0 \end{aligned}$$

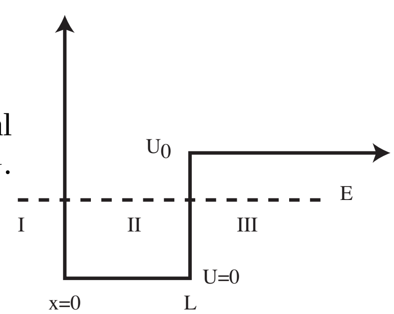
using the definite integral provided on the equation sheet. The Bohr radius is $r_n = a_0 n^2/Z = a_0$ for $n = 1$ and $Z = 1$. The true average radius $\langle r \rangle$ is larger than the Bohr radius.

4. A hydrogen atom is placed in a 6.0 T magnetic field. What's the mean energy for transitions from the $3d$ to $2p$ state, and the energy splitting due to the magnetic field?
Answer: There are two energy difference terms: the Bohr energy term, and the magnetic field splitting term. The Bohr energy difference term is $\Delta E = E_0(\frac{1}{2^2} - \frac{1}{3^2}) = E_0\frac{5}{36}$ or $13.6 \cdot 5/36 = 1.89$ eV. The magnetic field splitting terms go like

$$\Delta U = \Delta m_\ell \mu_B B = \Delta m_\ell \frac{9.274\times 10^{-24}\text{ J/T}}{1.602\times 10^{-19}\text{ J/eV}} \cdot 6.0\text{ T} = \Delta m_\ell 3.5\times 10^{-4}\text{ eV}$$

with selection rules saying $\Delta m_\ell = -1$ or +1.

5. Consider the potential shown at right. Describe the functional form of wavefunctions for the three regions ψ_I, ψ_{II}, and ψ_{III}. Show that there exist solutions at discrete energies.



Answer: Because the potential goes to ∞ at $x = 0$, we need to have $\psi_I = 0$. In region II we have to have $\psi_{II}(x=0) = 0$; beyond that, we have $U = 0$ so we just have traveling waves. Therefore we say that

$$\psi_{II} = A\sin(kx) \qquad \text{with} \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{\sqrt{2mE}}{\hbar}.$$

In region III ($x > L$) we are in a classically disallowed region so we will have an exponentially decaying wave function of the form

$$\psi_{III} = Be^{-\alpha(x-L)} \qquad \text{with} \qquad \alpha = \frac{\sqrt{2m(U-E)}}{\hbar}.$$

Our boundary conditions are

$$\psi_{II}(x=L) = \psi_{III}(x=L) \qquad \text{and} \qquad \frac{d\psi_{II}}{dx}|_{x=L} = \frac{d\psi_{III}}{dx}|_{x=L}$$

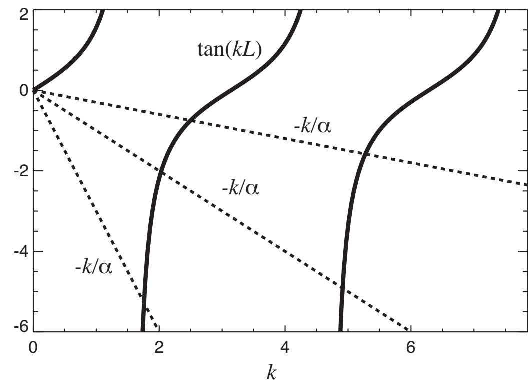
or

$$A\sin(kL) = Be^{-\alpha\cdot 0} = B \qquad \text{and} \qquad Ak\cos(kL) = -\alpha Be^{-\alpha\cdot 0} = -\alpha B$$

and if we divide these two equations we have

$$\frac{A\sin(kl)}{Ak\cos(kl)} = \frac{B}{-\alpha B} \qquad \Rightarrow \qquad \tan(kL) = -\frac{k}{\alpha}.$$

Viewing this as a graphical solution of how $\tan(kL)$ scales with k, versus how $-k/\alpha$ scales with k, gives



As you can see, no matter what slope $-1/\alpha$ we pick, we're going to have only discrete points where $\tan(kL)$ intersects with $-k/\alpha$, and thus discrete energy solutions.