Physics 251 exam 1, March 10, 2008. You may use a calculator, and the equation sheet that I have provided.

1. As a still-lurid 80-year old in 2061, Paris Hilton drives a Hummer H34 spaceship which is 12 m long. Al Gore, long since retired but still kicking around, drives a Honda CivicsLesson spaceship which is only 4 m long. Paris and Al both fly past you while you're standing on earth, and it looks to you like they're both driving spaceships of the same length. You know that Al drives at, but not even a smidgen above, the posted speed limit of $v = 0.55c$. How fast is Paris going relative to you on earth? Relative to Al?
Answer: In their respective frames, Paris' Hummer has a proper length of $L_{1,0} = 12$ m while Al's CivicsLesson has proper length of $L_{2,0} = 4$ m. We also see a speed of $\beta_2 = 0.55$ for Al (giving $\gamma_2 = 1.20$). We see the same length in our frame, or

$$\begin{aligned} L'_1 &= L'_2 \qquad \text{giving} \qquad \frac{L_{1,0}}{\gamma_1} = \frac{L_{2,0}}{\gamma_2} \\ \gamma_1 &= \frac{L_{1,0}}{L_{2,0}}\gamma_2 = \frac{12}{4} \cdot 1.20 = 3.60. \end{aligned}$$

Now from $\gamma = 1/\sqrt{1-\beta^2}$ we can find $\beta = \sqrt{1 - 1/\gamma^2}$ so Paris' speed is $\beta_1 = \sqrt{1 - 1/(3.60)^2} = 0.96$ as viewed by us on the ground. We now need to shift by a velocity of $0.55c$ from our frame to Al's frame to see how Al perceives the speed of Paris' Hummer H34, so that we have $v_1 = 0.96c$ (what we see for Paris) and $v = 0.55c$ (the velocity needed to shift into Al's frame), or

$$v_2 = \frac{v_1 - v}{1 - vv_1/c^2} = \frac{0.96c - 0.55c}{1 - 0.55 \cdot 0.96} = 0.87c.$$

2. X rays with an energy of 200 keV undergo Compton scattering from a target. At $30°$ scattering angle, find the energy of the Compton scattered X rays and the kinetic energy of the recoiling electron.
Answer: The Compton wavelength shift is

$$\lambda' - \lambda_0 = \frac{hc}{mc^2}(1 - \cos\theta) = \frac{1240 \text{ eV} \cdot \text{nm}}{511 \times 10^3 \text{ eV}}(1 - \cos 30°) = 3.25 \times 10^{-4} \text{ nm}$$

Let's get the energy of the scattered photon:

$$\begin{aligned} \lambda' &= \lambda_0 + (\lambda' - \lambda_0) \\ \frac{hc}{\lambda'} &= \frac{hc}{\lambda_0 + (\lambda' - \lambda_0)} \\ E' &= \frac{1}{\lambda_0/hc + (\lambda' - \lambda_0)/hc} = \frac{1}{1/E_0 + (\lambda' - \lambda_0)/hc} \\ E' &= \frac{1}{1/200 \times 10^3 \text{ eV} + (3.25 \times 10^{-4} \text{ nm})/(1240 \text{ eV} \cdot \text{nm})} = 190 \times 10^3 \text{ eV} \end{aligned}$$

or 190 keV. The kinetic energy of the scattered electron is then 200-190=10 keV.

3. When a metal is illuminated with a $\lambda = 400$ nm light, a retarding potential of 1.70 Volts makes the photocurrent go to zero. What's the work function of the metal? What retarding potential would be needed to stop the photocurrent if $\lambda = 300$ nm light were used?
   Answer: The stopping potential is the voltage needed to exactly compensate for the kinetic energy of the electrons, so $E_k = 1.70$ eV with $\lambda = 400$ nm, giving
   $$\varphi = \frac{hc}{\lambda} - E_k = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} - 1.70 \text{ eV} = 1.4 \text{ eV}$$
   for the work function. With $\lambda = 300$ nm light, we have
   $$E_k = \frac{hc}{\lambda} - \varphi = \frac{1240 \text{ eV} \cdot \text{nm}}{300 \text{ nm}} - 1.4 \text{ eV} = 2.73 \text{ eV}$$
   so that's the stopping potential needed to turn off the photocurrent.

4. A resurgent Russia has developed a fighter jet that can fly at Mach 50 (that is, $50\times$ the speed of sound; the speed of sound in air is about 300 m/s). As a CIA spy, you secretly measure its length on the ground to be 10 m. How much shorter does it appear to be when it flies past you at top speed? Use mathematical smarts to get an accurate numerical result.
   Answer: We have $L' = L_0/\gamma$, so
   $$L_0 - L' = L_0(1 - 1/\gamma).$$
   Now $\beta = (50 \cdot 300 \text{ m/s})/(3 \times 10^8 \text{ m/s})$ or $\beta = 5.0 \times 10^{-5}$. Therefore we should use a small β approximate form of $(1 - 1/\gamma)$:
   $$1 - \frac{1}{\gamma} = 1 - \sqrt{1 - \beta^2} = 1 - (1 - \beta^2)^{1/2} \simeq 1 - (1 - \frac{1}{2}\beta^2) = \frac{1}{2}\beta^2$$
   so that we have
   $$L_0 - L' = L_0(1 - 1/\gamma) \simeq L_0 \frac{1}{2}\beta^2 = (10 \text{ m}) \frac{1}{2}(5.0 \times 10^{-5})^2 = 1.25 \times 10^{-8} \text{ m}$$
   which is a very small distance...

5. In a quest to win either a Darwin Award or a Nobel Prize, you decide that you want to build a particle accelerator in your dorm. You use unshielded terminals to hook up an accelerating voltage of 200 kiloVolts, and also build a 0.2 Telsa magnet using soldered paper clips for the windings. If it is an electron that you accelerate, what is its kinetic energy? Momentum? Radius of curvature in the magnetic field?
   Answer: The energy given to an electron is qV so its kinetic energy is 200 keV and the rest mass of an electron is 511 keV/c^2 or 9.11×10^{-31} kg. We can find the momentum from
   $$\begin{aligned} E_k &= (\gamma - 1)mc^2 \quad &\Rightarrow \quad \gamma = 1 + \frac{E_k}{mc^2} = 1 + \frac{200 \text{ keV}}{511 \text{ keV}} = 1.391 \\ \gamma &= (1 - \beta^2)^{-1/2} \quad &\Rightarrow \quad 1 - \beta^2 = 1/\gamma^2 \\ \Rightarrow \quad \beta &= \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/(1.391^2)} = 0.695 \\ p &= \gamma m v = \gamma\beta mc^2/c = (1.391)(0.695)(511 \text{ keV})/c = 494 \text{ keV}/c. \end{aligned}$$

The radius of curvature is found from setting the Lorentz force equal to the centripetal force needed to maintain uniform circular motion:

$$\begin{aligned} qvB &= \gamma m \frac{v^2}{r} \\ r &= \frac{\gamma m v^2}{qvB} = \frac{\gamma m v}{qB} \\ &= \frac{\gamma m \beta c}{qB} = \frac{(1.39)(9.11 \times 10^{-31}\ \text{kg})(0.695)(3.00 \times 10^{8}\ \text{m/s})}{(1.602 \times 10^{-19}\ \text{C})(0.2\ \text{T})} = 8.24 \times 10^{-3}\ \text{m} \end{aligned}$$

or 8.24 mm.