Physics 251 exam 1, Feb. 19, 2007. You may use a calculator, and the equation sheet that I have provided.

1. Dick and Jane are each carrying an exact meter stick (according to them in their frame of reference). Dick goes past you at mach ten or 3000 m/s, while Jane goes past you at half the speed of light. How much shorter than 1 meter does Jane's meter stick appear to you? How much shorter than 1 meter does Dick's meter stick look to you?
Answer: In both cases, you see a length $\ell = \ell_0/\gamma$ relative to what they see, or a length difference of $\ell_0 - \ell = \ell_0(1 - 1/\gamma)$. In the case of Jane, we have $\beta = 0.5$ and $\gamma = 1/\sqrt{1-(1/2)^2} = 2/\sqrt{3}$ giving
$$\ell_0 - \ell = \ell_0(1 - \frac{1}{\gamma}) = (1\text{ m})(1 - \frac{\sqrt{3}}{2}) = 0.134\text{ meters.}$$
In the case of Dick, we have $\beta = (v/c) = (3\times 10^3/3\times 10^8) = 10^{-5}$ so we have to use a low-β expansion for the Lorentz factor γ:
$$\begin{aligned} \ell_0 - \ell &= \ell_0(1 - \frac{1}{\gamma}) = \ell_0\left(1 - (1-\beta^2)^{1/2}\right) \\ &\simeq \ell_0\left(1 - (1 - \frac{1}{2}\beta^2)\right) = \ell_0\frac{1}{2}\beta^2 \\ &= (1\text{ m})\frac{1}{2}(10^{-5})^2 = 5\times 10^{-10}\text{ m.} \end{aligned}$$
Given that atoms have a size of around 2×10^{-10} m, this is going to be hard to detect!

2. When a metal is illuminated with a $\lambda = 450$ nm light, a retarding potential of 1.50 Volts makes the photocurrent go to zero. What's the work function of the metal? What retarding potential would be needed to stop the photocurrent if $\lambda = 350$ nm light were used?
Answer: The stopping potential is the voltage needed to exactly compensate for the kinetic energy of the electrons, so $E_k = 1.50$ eV with $\lambda = 450$ nm, giving
$$\varphi = \frac{hc}{\lambda} - E_k = \frac{1240\text{ eV}\cdot\text{nm}}{450\text{ nm}} - 1.50\text{ eV} = 1.26\text{ eV}$$
for the work function. With $\lambda = 350$ nm light, we have
$$E_k = \frac{hc}{\lambda} - \varphi = \frac{1240\text{ eV}\cdot\text{nm}}{350\text{ nm}} - 1.26\text{ eV} = 2.28\text{ eV}$$
so that's the stopping potential needed to turn off the photocurrent.

3. While rushing to arrive nice and early for your 8:20 am recitation section so that you make your professor exceedingly happy, you get pulled over by a police officer for running a red ($\lambda = 600$ nm) light. You tell the police officer that it looked green ($\lambda = 500$ nm) to you. The police officer responds by adding a speeding ticket to your list of offences. What speed does the police officer write down on the ticket?

Answer: Let's define $A = \lambda_0/\lambda = \nu/\nu_0$. The relativistic Doppler shift for an approaching source involves $\theta = 180°$ is

$$\begin{aligned}
\nu &= \nu_0 \frac{1}{\gamma[1+\beta\cos\theta]} = \nu_0 \frac{1}{\gamma[1-\beta]} = \nu_0 \frac{\sqrt{1-\beta^2}}{1-\beta} \\
A &= \frac{\sqrt{(1-\beta)(1+\beta)}}{1-\beta} = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \\
A^2 &= \frac{1+\beta}{1-\beta} \\
A^2 - \beta A^2 &= 1+\beta \\
\beta &= \frac{A^2-1}{A^2+1} = \frac{(600/500)^2-1}{(600/500)^2+1} = 0.180
\end{aligned}$$

giving $v = \beta c = (0.180) \cdot 2.99 \times 10^8$ m/sec or 5.38×10^7 m/sec.

4. An electron curves with a radius of 6.0 mm in a magnetic field of 0.20 T. Calculate the speed, kinetic energy, total energy, and momentum of the electron.
   Answer: to maintain circular motion we have to have the Lorentz force provide the necessary centripetal force, or

$$\begin{aligned}
qvB &= \gamma m \frac{v^2}{r} \\
\gamma\beta &= \frac{qBrc}{mc^2} = \frac{(1.602 \times 10^{-19}\ \text{C}) \cdot (0.2\ \text{T}) \cdot (6.0 \times 10^{-3}\ \text{m}) \cdot (2.99 \times 10^8\ \text{m/s})}{(511 \times 10^3\ \text{eV}) \cdot (1.602 \times 10^{-19}\ \text{J/eV})} = 0.70.
\end{aligned}$$

Given $x \equiv \gamma\beta$, we can find β:

$$\begin{aligned}
x^2 &= \gamma^2\beta^2 = \frac{\beta^2}{1-\beta^2} \\
x^2 - x^2\beta^2 &= \beta^2 \\
\beta^2(1+x^2) &= x^2 \\
\beta &= \frac{x}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+1/x^2}} = \frac{1}{\sqrt{1+1/(\gamma\beta)^2}} = \frac{1}{\sqrt{1+1/(0.70)^2}} = 0.573
\end{aligned}$$

This in turn gives $\gamma = 1/\sqrt{1-(0.573)^2} = 1.22$. The total energy is $E = \gamma mc^2 = (1.22)(511\ \text{keV}) = 623$ keV. The kinetic energy is $E_k = (\gamma - 1)mc^2 = (0.22)(511\ \text{keV}) = 112$ keV. The momentum is

$$p = \gamma m_0 v = (\gamma\beta)\frac{m_0 c^2}{c} == (0.70) \cdot \frac{511\ \text{keV}}{c} = 358\ \text{keV}/c$$

5. A 20.000 keV photon undergoes Compton scattering, with the scattered photon coming out at an angle of $\theta = 25°$ relative to the incident photon. At what angle does the scattered

electron travel?
Answer: Let's first solve for the energy E' of the scattered photon:

$$\begin{aligned}
\lambda' - \lambda_0 &= \frac{hc}{mc^2}(1 - \cos\theta) \\
\frac{hc}{E'} &= hc\left(\frac{1}{E_0} + \frac{1 - \cos\theta}{mc^2}\right) \\
E' &= 1/\left(\frac{1}{E_0} + \frac{1 - \cos\theta}{mc^2}\right) = 1/\left(\frac{1}{20.000\text{ keV}} + \frac{1 - \cos 25^\circ}{510.9999\text{ keV}}\right) = 19.927\text{ keV}
\end{aligned}$$

That means the electron carries a kinetic energy of $E_k = 20.00 - 19.927 = 0.073$ keV or 73 eV, which is surely nonrelativistic. We can therefore say that the electron's momentum can be found from $E_k = (1/2)mv^2 = p_e^2/2m$ or

$$p_e = \sqrt{2mE} = \frac{\sqrt{2mc^2 E}}{c} = \frac{\sqrt{2 \cdot (510.9999\text{ keV}) \cdot (0.730\text{ keV})}}{c} = 8.637\text{ keV}/c.$$

Assuming that the incident photon traveled in the $\hat{x}$ direction, the $\hat{y}$ component of this momentum is equal and opposite to the $\hat{y}$ component of the scattered photon momentum, giving

$$\begin{aligned}
p_e \sin\phi &= p_{\lambda'} \sin\theta \\
\phi &= \arcsin\left(\frac{(19.927\text{ keV}/c) \cdot \sin(25^\circ)}{8.637\text{ keV}/c}\right) = 77^\circ.
\end{aligned}$$