Physics 251 exam 1, October 12, 2006.

1. An electron is accelerated through a potential difference of 100 kV. What's its speed? Kinetic energy? Momentum?
   Answer: The electron's kinetic energy is $(\gamma - 1)m_e c^2 = 100$ keV, so

$$\gamma = 1 + \frac{100 \text{ keV}}{m_e c^2} = 1 + \frac{100 \text{ keV}}{511 \text{ keV}} = 1.196$$

   and then we can find the speed from

$$\begin{aligned} \gamma^2 &= \frac{1}{1-\beta^2} \\ 1-\beta^2 &= \frac{1}{\gamma^2} \\ \beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/(1.196)^2} = 0.548 \end{aligned}$$

   or $v = \beta c = 1.65 \times 10^8$ m/s. The momentum is

$$p = \gamma m v = \gamma m c^2 \frac{\beta}{c} = (1.196)(511 \text{ keV})\frac{0.548}{c} = 335 \text{ keV/c}$$

   or in mks units $(1.196)(9.11 \times 10^{-31})(1.65 \times 10^8) = 1.80 \times 10^{-22}$ kg·m/s.

2. You tune your radio in your car to your favorite radio station which plays all Barry Manilow all the time: 88.1 MHz. However, you find instead that you are receiving the death metal station WIKD which broadcasts at 106.1 MHz. How fast are you going, and in which direction relative to WIKD?
   Answer: The source is emitting at $\nu_0 = 106.1$ MHz because you're hearing to WIKD even though you set your radio to 88.1 MHz. Therefore the frequency you receive is red-shifted ($\nu = 88.1$ MHz) which means you are traveling away from the source (or the source is receding from you), so that this is a Doppler shift problem with $\theta = 0°$ and $\cos\theta = 1$ giving

$$\begin{aligned} \nu &= \frac{\nu_0}{\gamma(1+\beta)} = \nu_0 \frac{\sqrt{1-\beta^2}}{1+\beta} = \nu_0 \frac{\sqrt{(1+\beta)(1-\beta)}}{1+\beta} = \nu_0 \sqrt{\frac{1-\beta}{1+\beta}} \\ (\frac{\nu}{\nu_0})^2 &= \frac{1-\beta}{1+\beta} \\ (\frac{\nu}{\nu_0})^2(1+\beta) &= 1-\beta \\ \beta\left(1+(\frac{\nu}{\nu_0})^2\right) &= 1-(\frac{\nu}{\nu_0})^2 \\ \beta &= \frac{1-(\nu/\nu_0)^2}{1+(\nu/\nu_0)^2} = \frac{1-(88.1/106.1)^2}{1+(88.1/106.1)^2} = 0.184 \end{aligned}$$

   so the person is traveling at 0.184 times the speed of light, away from station WIKD.

3. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum energy of 4.00 eV. What is the work function of the metal? What is the wavelength λ?
Answer: We have the following two equations:

$$E_1 = \frac{hc}{\lambda} - \varphi \qquad \text{and} \qquad E_2 = \frac{hc}{\lambda/2} - \varphi$$

with $E_1 = 1.00$ eV and $E_2 = 4.00$ eV. If we multiply the first equation by 2 and take the difference between the two equations we have

$$\begin{aligned} 2E_1 - E_2 &= (\frac{hc}{\lambda/2} - 2\varphi) - (\frac{hc}{\lambda/2} - \varphi) \\ 2E_1 - E_2 &= -\varphi \\ \varphi &= E_2 - 2E_1 = 4.00 - 2 \cdot 1.00 = 2.00 \text{ eV} \end{aligned}$$

and then we can find λ from

$$\begin{aligned} E_1 &= \frac{hc}{\lambda} - \varphi \\ E_1 + \varphi &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E_1 + \varphi} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.00 + 2.00 \text{ eV}} = 413 \text{ nm} \end{aligned}$$

4. You wish to excite a He^{+1} ion from the $n = 2$ state to the $n = 4$ state by absorption of first one photon and then another. What wavelengths should be used for the two different photons?
Answer: This is a one-electron ion with $Z = 2$ so the Bohr formula works just fine. The energies of the various states are

$$E_2 = -(13.6 \text{ eV})\frac{2^2}{2^2} = -13.6 \text{ eV}$$

and $E_3 = -6.04$ eV and $E_4 = -3.40$ eV. The two photons should have wavelengths of

$$\lambda_{\text{first}} = \frac{hc}{(-6.04) - (-13.6)} = \frac{1240 \text{ eV} \cdot \text{nm}}{7.56 \text{ eV}} = 164 \text{ nm} \qquad \text{and} \qquad \lambda_{\text{second}} = \frac{1240}{2.64} = 470 \text{ nm}$$

5. A smug city slicker bets a farmer that he can't get his 10 m long ladder into a 8 m long shed. The farmer, who reads Einstein each day after milking his cows, takes him up on the bet. He tells the city slicker to stand to the side of the shed and look in the windows at each end, and the farmer then runs fast as he can through the shed while carrying the ladder. How fast does the farmer have to run to win the bet? While on the run, how long does the shed appear to

the farmer, and does the farmer ever think his ladder is entirely inside the shed? (3 answers required).

Answer: For the city slicker in the stationary frame S_1 to observe the ladder as being completely inside the shed while the farmer runs by in the farmer's frame S_2, we must have a Lorentz contraction calculated from $L_1 = (1/\gamma)L_2$. Therefore

$$\gamma = \frac{L_2}{L_1} = \frac{10 \text{ m}}{8 \text{ m}} = \frac{5}{4}.$$

Now $\gamma^2 = 1/(1 - \beta^2)$ so

$$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 4^2/5^2} = \sqrt{9/25} = 3/5$$

or $v = 0.60c$. Now, to the farmer the shed appears to be moving toward him at $v = 0.60c$, so its length is Lorentz contracted according to $\gamma = 5/4$, giving an apparent shed length of $(8 \text{ m})/(5/4) = 6.4$ m. As a result, the farmer never sees the ladder as being completely inside the shed! But the farmer knows what the city slicker sees, and collects on the bet...