

## PHY 251, Spring 2008: Equations for Exam 2

This is the April 18, 2008 version.

$p = h/\lambda$ ,  $E = hc/\lambda = h\nu$  with  $hc = 1240 \text{ eV}\cdot\text{nm}$ .  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ .

$h = 4.14 \times 10^{-15} \text{ eV}\cdot\text{sec}$ ,  $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{sec}$ ,  $\hbar = h/(2\pi)$ , and  $c = 3.00 \times 10^8 \text{ m/sec}$ .

Masses:  $m_e = 511 \text{ keV}/c^2$ ,  $m_p = 939 \text{ MeV}/c^2$ .  $\Delta E \Delta t \geq \hbar/2$ ,  $\Delta x \Delta p \geq \hbar/2$ .

Bohr model:  $r_n = \frac{n^2}{Z} a_0$  with  $a_0 = \frac{\epsilon_0 \hbar^2}{m\pi e^2} = 0.053 \text{ nm}$ ,

and  $E_n = -\frac{Z^2}{n^2} E_0$  with  $E_0 = \frac{me^4}{8\epsilon_0^2 \hbar^2} = 13.60 \text{ eV}$ .

$m_r = \frac{m_1 m_2}{m_1 + m_2} \cdot \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = m \frac{v^2}{r}$ .

$-\frac{\hbar^2}{2m} \nabla^2 \psi + U\psi = i\hbar \frac{\partial \psi}{\partial t} = E\psi$  if  $U$  is time independent.

3D Schrödinger equation with Coulomb potential:  $\psi(r, \theta, \varphi) = R_{n,l}(r) \Theta_{l,m_l}(\theta) \Phi_{m_l}(\varphi)$ .

Volume element:  $r^2 dr \sin \theta d\theta d\varphi$ .

For  $U$  constant:

$E > U$ :  $\psi = A \sin kx + B \cos kx$ ,  $k = \sqrt{2m(E - U)}/\hbar$ , and

$E < U$ :  $\psi = C \exp[-ax]$ ,  $a = \sqrt{2m(U - E)}/\hbar$ .

Infinite well:  $E_n = n^2 \pi^2 \hbar^2 / (2mL^2)$  with  $\psi = \sqrt{2/L} \sin(n\pi x/L)$

Harmonic oscillator:  $E_n = (n + \frac{1}{2})\hbar\omega$ ,  $\psi_0 = A \exp[-\omega m x^2 / (2\hbar)]$ .

$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

Energy order of shells:  $1s < 2s < 2p < 3s < 3p < 4s \lesssim 3d < 4p < 5s < 4d < 5p$   
 $< 6s < 4f \lesssim 5d < 6p < 7s < 6d \lesssim 5f \dots$

$|L| = \sqrt{\ell(\ell + 1)}\hbar$ ,  $L_z = m_\ell \hbar$ ,  $\vec{\mu}_L = -(e/2m)\vec{L}$ .

$|S| = \sqrt{s(s + 1)}\hbar$ ,  $S_z = m_s \hbar$ ,  $\vec{\mu}_s = -(e/m)\vec{S}$ .

$U = m_\ell \mu_B B = 2m_s \mu_B B$  with  $\mu_B = e\hbar/(2m) = 9.274 \times 10^{-24} \text{ J/T}$ .

$\lambda_s - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta)$

$2d \sin \theta = n\lambda$

$$\int x^m e^{ax} dx = e^{ax} \sum_0^m (-1)^r \frac{m! x^{m-r}}{(m-r)! a^{r+1}} \quad \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$
$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}} \quad \int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$
$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} \quad \int \cos^2(ax) dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$