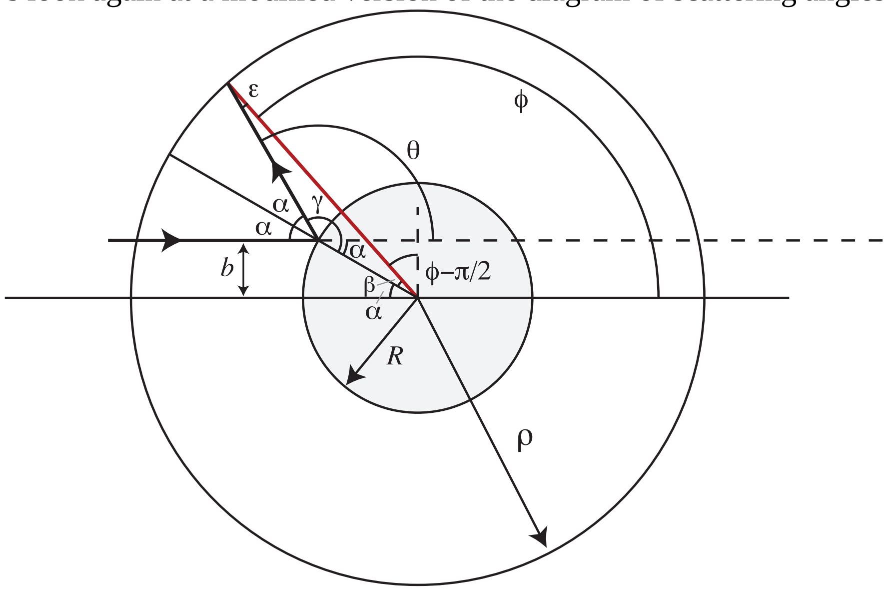
For Lab 7, let's look again at a modified version of the diagram of scattering angles:



What you are able to measure in lab are the following quantities:

- Δb is your increment in the impact parameter b, and from it you can get $dN/db = 1/(\Delta b)$ because you have one measurement per Δb.
- ϕ is the angle where you detect an "event" (the beam location as a result of a certain impact parameter b).
- $\Delta\phi$ is the resulting increment in detected angle ϕ as a result of incrementing b.
- ρ is the radius of the measuring apparatus.

What you'd like to know is not $dN/d\phi$ but $dN/d\theta$ and also θ, so that you can recover the radius R of the scatterer from

$$\frac{dN}{d\theta} = \frac{R}{2}\sin(\frac{\theta}{2})\,\frac{dN}{db} \qquad \Rightarrow \qquad R = 2\frac{dN/d\theta}{\sin(\theta/2)\,dN/db}. \tag{1}$$

Our problem in doing the analysis is that we've measured ϕ rather than the desired angle θ.

To consider the difference between ϕ and θ, let's define the small angular error ϵ as shown in the figure shown above. In the limit that $R \ll \rho$, we can say that

$$\theta \simeq \phi - \epsilon. \tag{2}$$

Now we need to do some geometry. Start by considering the 90° angle at upper left from the center of the image; from it, we have

$$\alpha + \beta + (\phi - \frac{\pi}{2}) = \frac{\pi}{2} \qquad \Rightarrow \beta = \pi - \alpha - \phi. \tag{3}$$

We can also find the angle γ from

$$\gamma + \alpha = \pi \qquad \Rightarrow \qquad \gamma = \pi - \alpha, \tag{4}$$

and we can then use the interior angle sum rule to find

$$\begin{aligned} \beta + \gamma + \epsilon &= \pi \\ \epsilon &= \pi - \beta - \gamma = \pi - (\pi - \alpha - \phi) - (\pi - \alpha) \\ &= 2\alpha + \phi - \pi. \end{aligned} \tag{5}$$

Now because $b = R \sin \alpha$, for small values of b we can make the approximation $b \simeq R\alpha$ and thereby substitute $\alpha \simeq b/R$ in Eq. 5. We have therefore arrived at an expression for ϵ of

$$\epsilon \simeq (\phi - \pi) + 2\frac{b}{R}. \tag{6}$$

Now you might say that we've achieved precisely nothing, because we need to know what the scatterer size R is in order to find the correction to get θ from ϕ in order to use data on θ to get $R\ldots$ And you're right, it is a circular train of logic. However, we can at least iteratively approach a self-consistent solution by noticing that the correction angle ϵ is smallest when $b \ll R$. Therefore, we can use the following procedure to solve for R from our scattering data:

Step 1. Take only the small b subset of your data [where ϵ is small and $\phi \simeq \theta$ and $(dN/d\phi) \simeq (dN/d\theta)$] and get a first estimate of R from Eq. 1 of

$$R_0 = 2\frac{dN/d\phi}{\sin(\phi/2)\, dN/db} \qquad \text{for } b \ll R.$$

Step 2. Using the first guess R_0 determined above, calculate ϵ for each value of ϕ using Eq. 6 of

$$\epsilon \simeq (\phi - \pi) + 2\frac{b}{R_0}.$$

Step 3. Now you can convert each value of ϕ into an estimate of θ from Eq. 2 of

$$\theta \simeq \phi - \epsilon.$$

From this set of θ data you can now determine $dN/d\theta$ instead of $dN/d\phi$, and you can now use your entire data set to calculate the next better guess of R as

$$R_1 = 2\frac{dN/d\theta}{\sin(\theta/2)\, dN/db}.$$

Step 4. Now if you wanted you could use this *better* guess of R given by R_1 to calculate a refined estimate of ϵ as per step 2, and then do step 3 again, and so on in an iterative procedure.

For the purposes of this lab, one pass through the above steps is sufficient (you don't need to iterate).