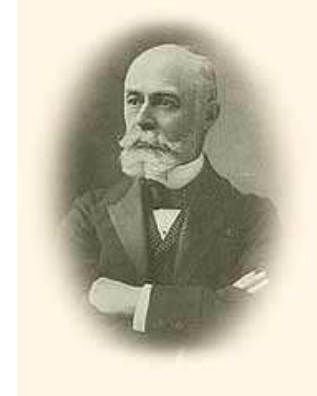


## *Review: the electron*

- Between Thomson and Millikin, we know its mass and its charge. We know that it's about  $1/2000^{th}$  the mass of a hydrogen atom.
- We have an estimate of the electron's size from comparing its  $E = m_e c^2$  energy with the energy required to bring its charge within a certain radius. This gives the classical radius of the electron, which is  $2.812 \times 10^{-15}$  m which is much smaller than atoms which have a size of about  $2 \times 10^{-10}$  m or 0.2 nm.
- Because gravity is wimpy yet it dominates our everyday experience, we know that matter is by and large electrically neutral. This all leads to Thomson's "plum pudding" model of the atom.
- But plum pudding can't be quite right! The resonant frequency of an electron at the center of a hydrogen-sized blob of pudding would be around  $2 \times 10^{15}$  Hz corresponding to steady light emission at  $\lambda = 120$  nm. In fact, atoms don't sit there radiating away in the ultraviolet as a matter of routine.

## *Meanwhile. . . Radioactivity!*

- Early 1896: Henri Becquerel noticed that uranium compounds would fog photographic plates—the discovery of radioactivity.
- 1898: Marie Sklodowska Curie measures radioactivity by looking at ionization of air. Unaffected by chemical binding, heat, etc.! Husband Pierre then joins research; they discover radium and polonium.
- Radioactive decay releases energies in the MeV range!!!
- Becquerel, and Marie and Pierre Curie share the 1903 Nobel Prize in Physics. Marie is awarded the 1911 Nobel Prize in Chemistry.



Antoine Henri Becquerel  
(1852–1908)



Marie Curie (1867–1934) and  
Pierre Curie (1859–1906)

# *Lots of energy!*

- Rutherford and Soddy, 1903: “The energy of radioactive change must therefore be at least twenty-thousand times, and may be a million times, as great as the energy of any molecular change.” *Remember that chemistry happens at  $\sim 3\text{--}10\text{ eV}$ , while nuclear decays happen at several MeV or  $10^6\text{ eV}$ .*
- Soddy in 1904: “If it [the energy of the nucleus] could be tapped and controlled what an agent it would be in shaping the world’s destiny! The man who put his hand on the lever by which a parsimonious nature regulates so jealously the output of this store of energy would possess a weapon by which he could destroy the earth if he chose.”
- Soddy continued: “The fact that we exist is a proof that [massive energetic release] did not occur; that it has not occurred is the best possible assurance that it never will. We may trust Nature to guard her secret.”

Quoted in Rhodes, *The making of the atomic bomb*

## *Other gems from [www.wikiquote.org](http://www.wikiquote.org)*

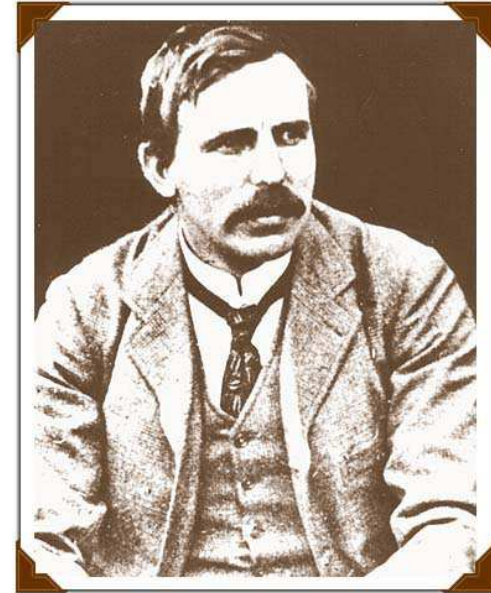
- “There is no likelihood man can ever tap the power of the atom.”—Robert Millikan, 1923.
- “There is not the slightest indication that nuclear energy will ever be obtainable. It would mean that the atom would have to be shattered at will.”—Albert Einstein, 1932.
- “The energy produced by the breaking down of the atom is a very poor kind of thing. Anyone who expects a source of power from the transformation of these atoms is talking moonshine.”—Ernest Rutherford, shortly after splitting the atom for the first time.
- “That is the biggest fool thing we have ever done [research on]. . . The bomb will never go off, and I speak as an expert in explosives.”—Admiral William Leahy, U.S. Admiral working in the U.S. Atomic Bomb Project, advising President Truman on atomic weaponry, 1944.
- “Nuclear-powered vacuum cleaners will probably be a reality in 10 years.”—Alex Lewyt, president of vacuum cleaner company Lewyt Corp., in the New York Times in 1955.

plus

- “It’s hard to make predictions—especially about the future.”—attributed to Robert Storm Petersen.

## *Let's start with Rutherford: the alpha male*

- Grew up on a farm in New Zealand, and studied at the University there. Applied for a graduate scholarship at Cambridge and worked at home while awaiting a reply. When the scholarship letter came (1894), he threw down his shovel and said “That’s the last potato I will ever dig.”
- McGill University in Montreal, 1898–1907.  
University of Manchester, 1907–1919. Cavendish Professor at Cambridge, 1919–1937.
- “All science is either physics or stamp collecting”—but look in what field he earned a Nobel prize. . .



Ernest Rutherford (1871–1937; Nobel Prize in Chemistry, 1908)

# Discovery of the alpha particle

Rutherford at McGill in Montreal, 1905:

- Fill thin-glass-wall tube with radon, which emits alpha particles ( $\alpha$ ).
- Surround that tube with another thick-walled, evacuated tube.
- After a few days, helium identified in the outer tube by its characteristic spectrum.
- Alpha particles are helium nuclei (2 protons, 2 neutrons, or  ${}^4_2\text{He}_2$ , using  ${}^A_Z\text{X}$ ).
- Other radioactive decay emissions:  $\beta^-$  are electrons,  $\gamma$  are very energetic photons.

1908 Nobel Prize in Chemistry. Rutherford's paper (1909?!) is reproduced [here](#).



Rutherford's apparatus (from the [Museum at the Cavendish Lab](#)).

## Use $\alpha$ particles to probe the nucleus

- Radium emits  $\alpha$  particles with 5 MeV kinetic energy. Assume for now (not quite right) that

$$m = 2 \cdot 938.3 + 2 \cdot 939.6 = 3757.8 \text{ MeV}/c^2; \text{ gives}$$

$$v = 2\sqrt{5 \text{ MeV}/(3757.8 \text{ MeV}/c^2)} = 0.365 c = 2.2 \times 10^7 \text{ m/s.}$$

- Equivalently,  $m = 2 \cdot 1.673 \times 10^{-27} + 2 \cdot 1.675 \times 10^{-27} = 6.696 \times 10^{-27} \text{ kg}$  and  
 $v = 2\sqrt{(5 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV})/(6.696 \times 10^{-27} \text{ kg})} = 2.2 \times 10^7 \text{ m/s.}$
- Gold nucleus significantly outweighs  $\alpha$ . Gold can be hammered into very thin foils.

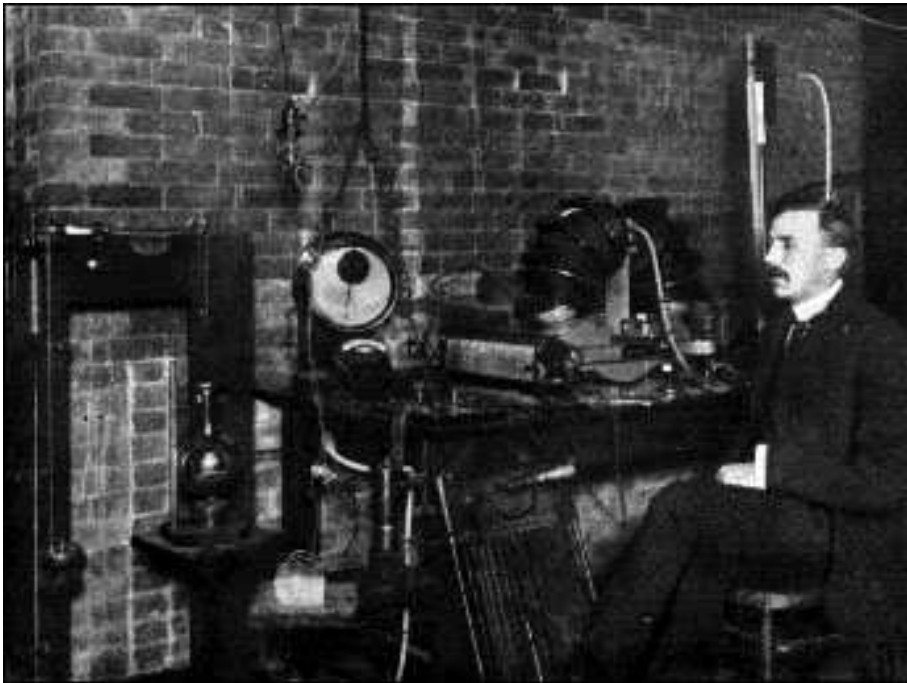
Interatomic spacing:

$$\left(\frac{A}{\rho N_A}\right)^{1/3} = \left(\frac{197 \text{ g/mol}}{(18.9 \text{ g/cm}^3) \cdot (6.02 \times 10^{23} \text{ mol}^{-1})}\right)^{1/3} = 2.6 \times 10^{-8} \text{ cm}$$

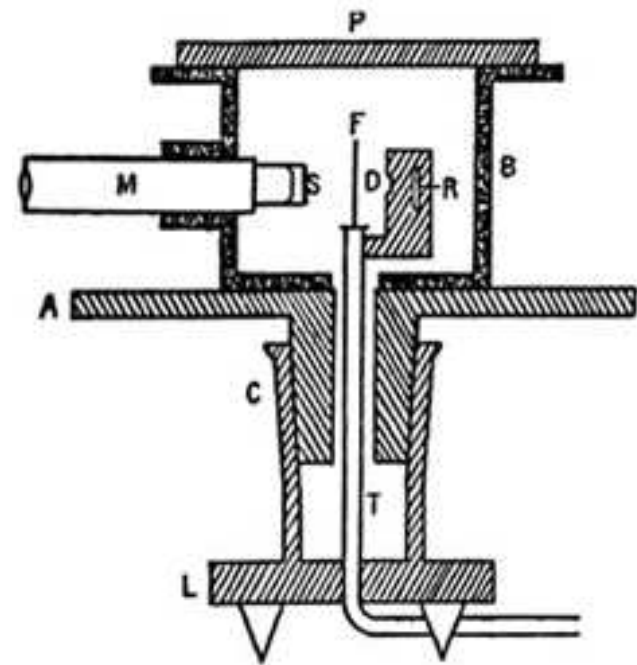
- While at McGill, Rutherford had noticed significant scattering at large angles. That seemed odd. . .
- After receiving Nobel Prize, Rutherford is offered position at Manchester in U.K. Assigns further investigation of this to Ernest Marsden (18 year old undergrad at start of experiments), aided by Rutherford's "postdoc" Hans Geiger.

# The experiment

Set the Radium  $\alpha$  emitter R and foil F to one angle. Sit in the dark for several minutes to become dark-adapted (Rutherford would tell stories). Stare into the microscope M and count flashes of light on the zinc sulfide screen S for a long time. . .



Rutherford at the setup



Sketch from the Marsden and Geiger paper

# Thomson scattering I

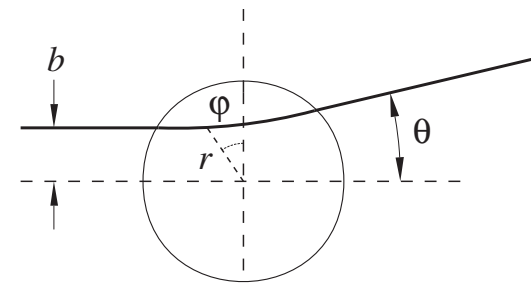
- What if the scattering were done by an atom-sized ball of uniform positive charge?
- Serway doesn't do this in detail. *Modern Physics* by Kenneth Krane (Wiley, 1996) does.
- Particle of charge  $ze$  approaches a sphere of charge  $Ze$  in a radius  $R$ , with an impact parameter (distance off from the centerline) of  $b$ .
- For small  $\theta$ , distance particle travels through sphere is chord  $c$  at  $b$ , or

$$b^2 + \left(\frac{c}{2}\right)^2 = R^2 \quad \text{or} \quad c = 2\sqrt{R^2 - b^2}.$$

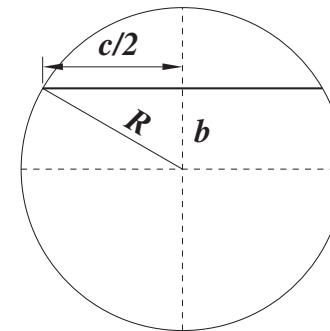
- Time that particle spends inside the sphere is (Krane Eq. 6.5)

$$T = 2\frac{\sqrt{R^2 - b^2}}{v}$$

Scattering geometry (like Krane Fig. 6.3):



Chord of sphere:



## Thomson scattering II

- What force does particle experience? In the last lecture, in the slide on “Indigestion from plum pudding,” we found (see Krane Eq. 6.1)

$$(1) \quad F = \frac{zZe^2}{4\pi\epsilon_0 R^3} r$$

where this time we have an  $\alpha$  particle with  $z = 2$  rather than an electron with  $z = 1$ . Thus

$$F_y = F \cos \varphi \simeq k z r \frac{b}{r} \simeq k z b \quad \text{with} \quad k \equiv \frac{Ze^2}{4\pi\epsilon_0 R^3}$$

- Momentum impulse transferred to the scattered particle (like Krane Eqs. 6.2, 6.6):

$$\Delta p_y = \int F_y dt = \int k z b dt = k z b T = \frac{2k z b}{v} \sqrt{R^2 - b^2}.$$

- Scattering angle  $\theta$  is then (Krane Eq. 6.8)

$$(2) \quad \theta \simeq \frac{\Delta p_y}{p} = \frac{2k z b}{m v^2} \sqrt{R^2 - b^2}.$$

# Thomson scattering III

- Again, scattering angle  $\theta$  is (our Eq. 2; Krane Eq. 6.8)

$$(3) \quad \theta \simeq \frac{\Delta p_y}{p} = \frac{2kzb}{mv^2} \sqrt{R^2 - b^2}.$$

- For  $b = R/2$  and using Eq. 1, we have (like Krane Eq. 6.9)

$$(4) \quad \theta_{\text{typ.}} = 2k \frac{z(R/2)}{E_k} R \sqrt{\frac{3}{4}} = \sqrt{3} \frac{Ze^2}{4\pi\epsilon_0 R^3} \frac{zR^2}{2E_k} = \frac{\sqrt{3} z Z e^2}{8\pi\epsilon_0 R E_k}$$

- Use  $z = 2$ ,  $Z = 79$ ,  $R = 2.6 \times 10^{-10}$  m,

$$E_k = (5 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV}) = 8.0 \times 10^{-13} \text{ J to obtain}$$

$$(5) \quad \begin{aligned} \theta_{\text{typ.}} &= \frac{\sqrt{3} z Z e^2}{8\pi\epsilon_0 R E_k} \\ &= \frac{\sqrt{3} \cdot 2 \cdot 79 \cdot (1.602 \times 10^{-19})^2}{8\pi \cdot (8.854 \times 10^{-12}) \cdot (2.6 \times 10^{-10}) \cdot (8.0 \times 10^{-13})} \\ &= 1.5 \times 10^{-4} \text{ rad} = 0.15 \text{ mrad} \end{aligned}$$

## Rutherford's suggestion

- So... What's inside the bowl of plum pudding? Let's shoot bullets with a mass  $\sim 8000\times$  times that of the electron to find out!
- Gold hammered to  $2\ \mu\text{m}$  thickness: about  $10^4$  atoms thick.
- Net deflection angle for  $N$  uncorrelated scatterings of value  $\theta_{\text{typ}}$ . each is  $\sqrt{N}\theta_{\text{typ}}$ .
- Thus for gold foil we expect about  $100\ \theta_{\text{typ}}$ , or about 15 mrad.
- Not at all consistent with Rutherford's earlier observations! Look into it more closely.
- Hence Rutherford's suggestion that Marsden (again, age 18 at the outset of this) and Geiger do some careful experiments to map out the angular dependence of  $\alpha$  particle scattering in detail.

## *The unexpected*

Marsden and Geiger find significant scattering at large angles, including back towards the  $\alpha$  particle source! Rutherford:

It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration I realised that this scattering backwards must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greatest part of the mass of the atom was concentrated in a minute nucleus.

As reported by Rhodes in *The making of the atomic bomb*.

Note: in 1906 the British Navy launched the *HMS Dreadnought* which was dramatically superior to all other warships of its day, with 12-inch guns. It was surpassed in 1913 by the *HMS Queen Elizabeth* with 15-inch guns (they could fire 870 kg shells with a muzzle velocity of 785 m/sec, reaching a range of 18 km!). Now you know what Rutherford had in mind!

# Rutherford's interpretation

- Return to Eq. 4 of

$$\theta_{\text{typ.}} = \frac{\sqrt{3} z Z e^2}{8\pi\epsilon_0 R E_k}$$

- We need to scale up from single scattering of 0.15 mrad to 1.5 rad, or a factor of  $10^4$ .
- What's the chance of having *all*  $10^4$  scattering events give  $\theta > \theta_{\text{typ.}}$ ? Assume probability of  $(1/2)$  for each scatter to be larger than  $\theta_{\text{typ.}}$ ; net probability is then  $(1/2)^{10^4}$ . If we have  $A = B^C$ , then

$$\log_{10} A = \log_{10} B^C = C \log_{10} B,$$

so  $(1/2)^{10^4} = 10^{10^4 \log_{10}(1/2)} = 10^{-3000}$ . You can imagine Rutherford saying, “Not bloody likely!”

- To do this by  $N$  uncorrelated scatters, we would need  $\sqrt{N} = 10^4$  or  $N = 10^8$  or a foil thickness of  $10^8 \cdot 2.6 \times 10^{-10} \text{ m} = 2.6 \text{ cm}$ . Ludicrous!
- What can we change?  $R$  is the only thing! Must scale it down by  $10^{-4}$ , from  $2.6 \times 10^{-10}$  to  $2.6 \times 10^{-14}$ .

## Re-examine the situation

- Conclusion: the nucleus is *very* small. When Rutherford figures it out (early 1911), he marches into Geiger's office humming "Onward Christian Soldiers" and announces, "I know what the atom looks like!"
- With a point-like nucleus, electrons must be in some sort of orbital motion.
- Now we re-think the problem, with  $R \ll b$ . It can be shown that a particle follows a hyperbolic path when it passes near a point source of a  $1/r^2$  repulsive force. For Coulomb repulsion, one obtains (Krane Eq. 6.12)

$$(6) \quad \frac{1}{r} = \frac{1}{b} \sin \varphi + \frac{zZe^2}{8\pi\epsilon_0 b^2 E_k} (\cos \varphi - 1),$$

where  $E_k$  is the kinetic energy of the alpha particle.

- For scattering angles of  $\theta = \pi - \varphi$ , one can solve for  $b$  to obtain (Krane Eq. 6.13)

$$(7) \quad b = \frac{zZe^2}{8\pi\epsilon_0 E_k} \cot\left(\frac{1}{2}\theta\right).$$

## Rutherford scattering II

- Fraction  $f$  of scatterings at an angle of  $\theta$  or larger will be given by the fractional area taken up by atoms with impact parameter  $b$  in a foil of thickness  $t$ ; this can be found to be (Krane Eq. 6.15)

$$(8) \quad f_{>\theta} = \frac{\rho N_A}{A} t \pi b^2.$$

- To find the range of impact parameters  $b$  to  $b + db$  which produce scattering within an angular range  $d\theta$ , we must find

$$(9) \quad d(f_{>\theta}) = \frac{N_A \rho}{A} t (2\pi b db).$$

- The  $db$  term can be found from Eq. 7 to be (Krane Eq. 6.16)

$$(10) \quad db = \frac{zZe^2}{8\pi\epsilon_0 E_k} \left(-\csc^2\left[\frac{1}{2}\theta\right]\right) \left(\frac{1}{2}d\theta\right),$$

which gives (Krane Eq. 6.17)

$$(11) \quad d(f_{>\theta}) = \frac{N_A \rho}{A} \pi t \left(\frac{zZe^2}{8\pi\epsilon_0 E_k}\right)^2 \csc^2\left(\frac{1}{2}\theta\right) \cot\left(\frac{1}{2}\theta\right) d\theta.$$

## Rutherford scattering III

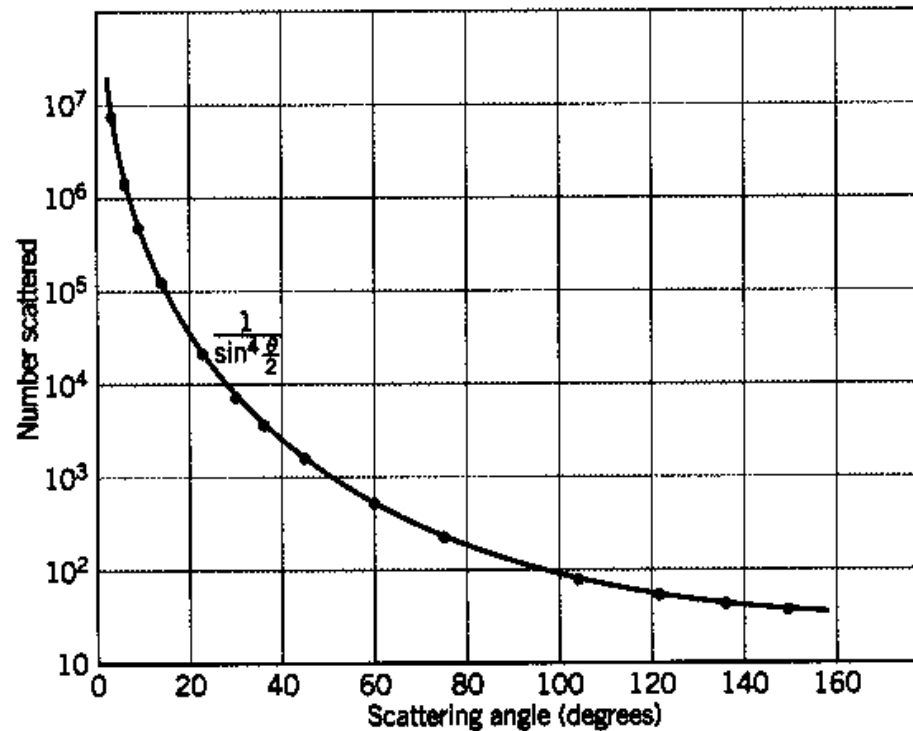
- In fact, our detector is not likely to collect *all* events at an angle larger than  $\theta$ ; instead, we will consider the fraction of events which fall in a ring at a distance  $r$  from the scatterer which collects an angular range of  $d\theta$  about  $\theta$ .
- The detector area integrated over  $2\pi$  azimuthally, the radius of the  $d\theta$  ring is  $r \sin \theta$ , and the width of the ring is  $r d\theta$ , so the detector area is  $2\pi r^2 \sin \theta d\theta$ .
- The fraction  $N(\theta)$  of events we expect to detect can then be shown to be (Krane Eq. 6.18)

$$(12) \quad N(\theta) = \frac{N_A \rho}{A} \frac{t}{4r^2} \left( \frac{zZe^2}{8\pi\epsilon_0 E_k} \right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}.$$

which is the expression Rutherford derived for the scattering distribution.

# Angular distribution

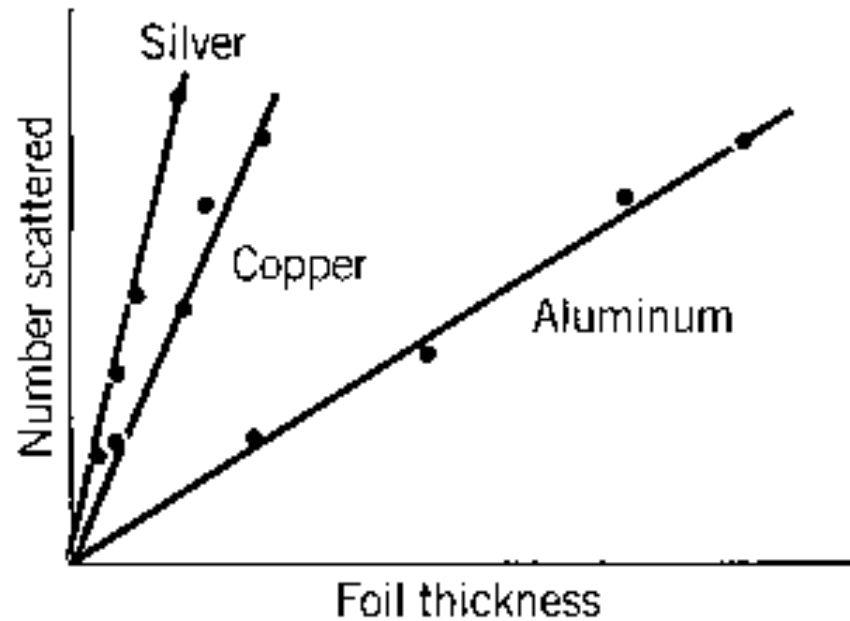
Agreement between Rutherford's theory and Marsden's experiments (Krane Fig. 6.14):



**FIGURE 6.14** The dependence of scattering rate on the scattering angle  $\theta$ , using a gold foil. The  $\sin^{-4}(\theta/2)$  dependence is exactly as predicted by the Rutherford formula.

## *Effect of foil thickness*

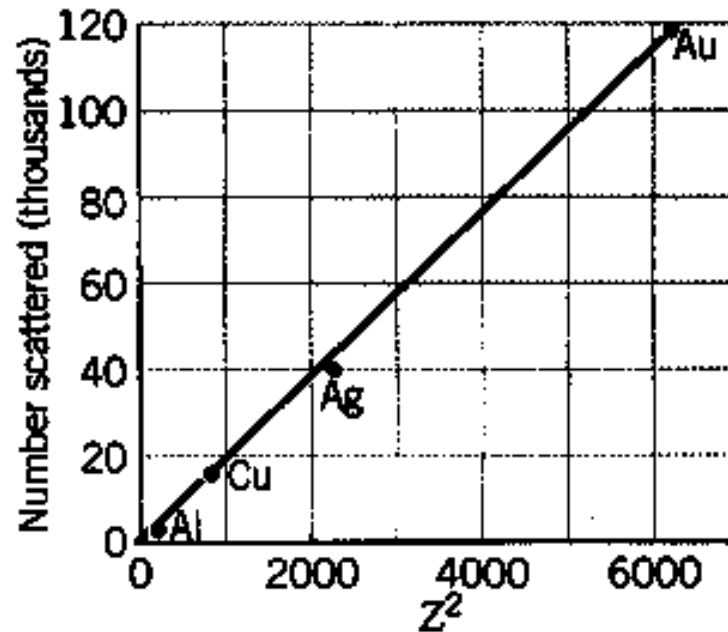
Rutherford predicts that  $N(\theta) \propto t$ , the foil thickness, whereas recall that the Thomson model would have predicted  $N(\theta) \propto \sqrt{t}$ . See Krane Fig. 6.11:



**FIGURE 6.11** The dependence of scattering rate on foil thickness for three different scattering foils.

## Effect of atomic number $Z$

Rutherford predicts  $N(\theta) \propto Z^2$ , which stands in contrast to the linear dependence on  $Z$  in Eq. 4. See Krane Fig. 6.12:



**FIGURE 6.12** The dependence of scattering rate on the nuclear charge  $Z$  for foils of different materials. The data are plotted against  $Z^2$ .

## *Distance of closest approach*

- In backscattering, the kinetic energy of the alpha particle must be converted completely into electrostatic potential energy at the point of closest approach.
- This gives a minimum radius of (Krane Eq. 6.22)

$$(13) \quad r_{\min} = \frac{1}{4\pi\epsilon_0} \frac{zZe^2}{E_k},$$

which for a 5 MeV alpha on gold gives  $r_{\min} = 5 \times 10^{-14}$  m.

- In fact, an absolute measurement of  $N(\theta)$  was shown to be consistent with a radius of the nucleus which is almost a tenth of this value, which is why the alpha particle does not induce fission in gold.

# The Rutherford atom I

- Rutherford model: electrons orbiting nucleus.
- Orbital frequency? Coulomb force provides centripetal force, or

$$(14) \quad m \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

from which we can calculate a velocity of  $v = 1.6 \times 10^6$  m/sec and a kinetic energy of  $1.2 \times 10^{-18}$  Joules or 7 eV.

- Oscillation frequency is

$$(15) \quad f = \frac{v}{2\pi r} = \sqrt{\frac{1}{2(2\pi)^3\epsilon_0} \frac{e^2}{mr^3}}.$$

If we use  $r = 10^{-10}$  m to represent atomic dimensions, we obtain  $f \simeq 3 \times 10^{15}$  Hz, which corresponds to UV light with a wavelength of about 100 nm.

- Again, we have to worry if this is reasonable because we don't see atoms radiating 100 nm light just while they're sitting there.

## The Rutherford atom II

- How long will the electron be able to orbit the atom? The acceleration can be found from

$$(16) \quad ma = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}.$$

- You'll learn that in classical electrodynamics, one can calculate the power radiated by an accelerated charge by the Larmor formula of

$$(17) \quad P = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^2 a^2}{c^3},$$

from which we find that the power radiated by the electron would be expected to be  $P \simeq 4 \times 10^{-9}$  J/sec.

- The electron of energy  $1.2 \times 10^{-18}$  J would therefore be expected to last only 0.3 nanoseconds before it would have radiated all its energy and crashed into the nucleus! Prof. Peterson in particular is older than 0.3 nanoseconds, so it's unreasonable to speak of a world where atoms last only that long. . .

## *Enter Niels Bohr*

- A very promising young physicist from Denmark.
- Spent September 1912 through May 1913 at the Cavendish Lab under J.J. Thomson.
- Met Rutherford who came to talk about his experiments in late 1912.
- Arranged a visit to Manchester; winds up spending April–July 1912 with Rutherford.
- Rutherford gives him the challenge of making sense of atoms with small nuclei.



Niels Bohr (1885–1962; Nobel Prize 1922)

# Bohr's model I

- The Bohr model is usually explained using electron waves, but that idea didn't come til 1924! We'll pick that up at the end.
- Bohr came about his model by indirect means: a feeling that Planck's constant  $h$  and quantization was really fundamental to how electrons behave in atoms and therefore must be involved.
- Start with the fundamentals: Coulomb force provides centripetal force for circular motion:

$$(18) \quad \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = m_e \frac{v^2}{r}$$

- Note the dimensions of  $4\pi\epsilon_0/e^2$ :  $\frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \frac{1}{\text{C}^2} = \frac{\text{sec}^2}{\text{kg m}^3}$ .
- Now  $h$  has dimensions Joules·sec or  $\text{kg} \cdot \text{m}^2/\text{sec}$ , so if we have  $\frac{4\pi\epsilon_0 h^2}{e^2}$  we must also divide by  $m_e$  to get kilograms to cancel out:  $\frac{\text{sec}^2}{\text{kg m}^3} \cdot \frac{\text{kg}^2 \text{m}^4}{\text{sec}^2} \cdot \frac{1}{\text{kg}} = \text{m}$ .

## Bohr's model II

- We therefore have an expression with dimensions in meters, with a length scale in the range of atomic dimensions!

$$(19) \quad \frac{4\pi\epsilon_0 h^2}{m_e e^2} = 2.1 \text{ nm},$$

- Put the result of Eq. 19 into the Coulomb-centripetal result of Eq. 18:

$$(20) \quad \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} = m_e \frac{v^2}{r}$$
$$Z \frac{m_e e^2}{4\pi\epsilon_0 h^2} r = Z \frac{r}{2.1 \text{ nm}} = \frac{r^2 m_e^2 v^2}{h^2} = \left( \frac{r m_e v}{h} \right)^2$$

- Angular momentum  $\vec{l} = \vec{r} \times \vec{p}$  is  $l = r m v$  in circular motion, so the right hand side of Eq. 20 is  $(l/h)^2$ , giving

$$(21) \quad Z \frac{r}{2.1 \text{ nm}} = Z \frac{m_e e^2}{4\pi\epsilon_0 h^2} r = \left( \frac{l}{h} \right)^2.$$

which is a tidy expression with radii  $r$  normalized to atomic dimensions, and angular momentum scaled by Planck's constant  $h$ .

## Bohr's model III

- Again, we have Eq. 21:  $Z \frac{r}{2.1 \text{ nm}} = \left(\frac{l}{\hbar}\right)^2$  using  $\frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 2.1 \text{ nm}$ .
- Hmm. . . Planck found success with  $E = nh\nu$ . Let's try

$$(22) \quad l = mvr = n \frac{h}{2\pi} = n\hbar$$

- We then have

$$(23) \quad Z \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} r = \left(\frac{l}{\hbar}\right)^2 = (2\pi)^2 \left(\frac{n\hbar}{\hbar}\right)^2$$
$$Z \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} r = n^2$$

- Solve for radii  $r_n$ :

$$(24) \quad r_n = \frac{n^2}{Z} \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{n^2}{Z} a_0 \quad \text{with} \quad a_0 \equiv \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 0.053 \text{ nm}.$$

## Bohr's model IV

- Again, we have  $r_n = \frac{n^2}{Z} a_0$  with  $a_0 \equiv \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.053$  nm being called the Bohr radius.
- This is startling and exciting. It says we that electrons have quantized radii beginning with 0.5 Å, which is a typical atomic dimension.
- If we know the radius  $r$ , we know the kinetic energy and thus the velocity  $v$ :

$$(25) \quad \begin{aligned} l = rmv &= n\hbar \\ v &= \frac{n\hbar}{m_e r} = \frac{n\hbar}{m_e} \frac{Zm_e e^2}{n^2 4\pi\epsilon_0 \hbar^2} = \frac{Z}{n} \frac{e^2}{2h\epsilon_0 c} c = \frac{Z}{n} \alpha c \end{aligned}$$

with a *fine structure constant* of  $\alpha \equiv \frac{e^2}{2\epsilon_0 hc} \simeq \frac{1}{137.036}$ .

- For  $Z = 1$  and  $n = 1$ , the speed is nonrelativistic:  $c/137$ . For larger atoms and orbital radii, relativity starts to enter in; it turns out relativistic corrections can be treated as a power series in  $\alpha$ .
- Note: people have tried to find theories for  $\alpha \equiv 1/137 \dots$

# Bohr energy I

- Since  $v \simeq c/137$ , we can use the classical result  $E_k = (1/2)mv^2$  and Eqs. 20 to find the kinetic energy:

$$(26) \quad E_k = \frac{1}{2}m_e v^2 = \frac{1}{2} r \left( \frac{m_e v^2}{r} \right) = \frac{1}{2} r \frac{1}{4\pi\epsilon_0} \frac{Z e^2}{r^2} = \frac{Z e^2}{8\pi\epsilon_0 r}.$$

- Calculate the electrostatic potential energy  $U$ , using the usual zero point at  $r \rightarrow \infty$ :

$$(27) \quad U = -\frac{Z e^2}{4\pi\epsilon_0 r}.$$

- Total energy is then

$$(28) \quad E = E_k + U = -\frac{Z e^2}{8\pi\epsilon_0 r}.$$

- Insert Bohr radius of Eq. 24:

$$(29) \quad E_n = -\frac{Z e^2}{8\pi\epsilon_0} \frac{Z}{n^2} \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} = -\frac{Z^2}{n^2} \frac{m_e e^4}{8h^2 \epsilon_0^2}$$

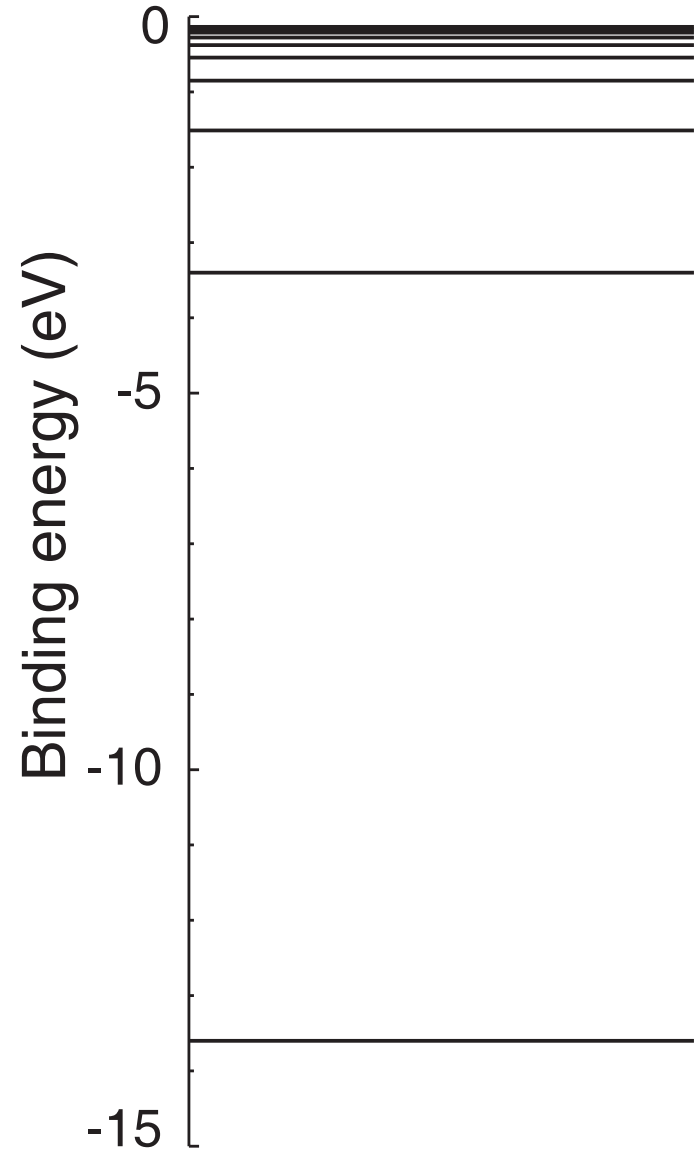
## Bohr energy II

The energy is quantized:

$$(30) \quad E_n = -\frac{Z^2 m_e e^4}{n^2 8h^2 \epsilon_0^2} = -\frac{Z^2}{n^2} E_0$$

with

$$E_0 \equiv \frac{m_e e^4}{8h^2 \epsilon_0^2} = 13.60 \text{ eV}$$



## *Does Bohr make sense?*

- Bohr's paper is [here](#).
- Bohr developed his basic ideas while at Manchester with Rutherford April-July 1912. Started writing a paper while on his honeymoon in August 1912, before starting to teach as an Assistant Professor in Copenhagen. Finally sent a draft of his paper to Rutherford in March 1913.
- Rutherford wrote to Bohr:

There appears to me one grave difficulty in your hypothesis, which I have no doubt you fully realize, namely, how does an electron decide what frequency it is going to vibrate at when it passes from one stationary state to the other? It seems to me that you would have to assume that the electron knows beforehand where it is going to stop.



Lord and Lady Rutherford, and Niels and Margrethe Bohr (probably ca. 1930)

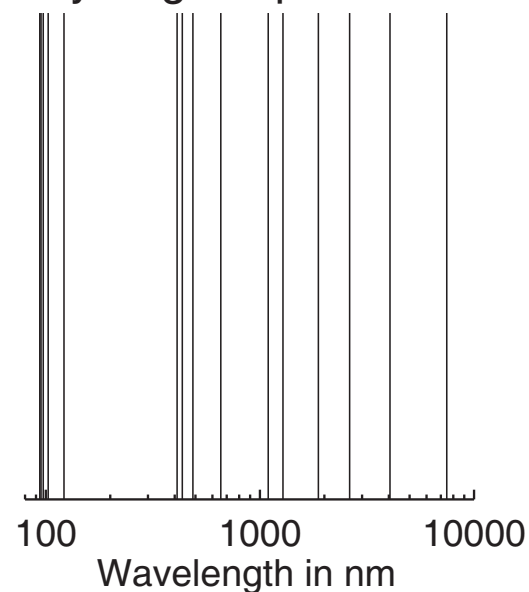
# Nonsense or not?

- While a student in 1913, Otto Stern (Nobel Prize 1946) made a vow with Max von Laue (Nobel Prize 1914): “If this nonsense of Bohr should in the end prove to be right, we will quit physics!”—according to B. Friedrich and D. Herschbach, *Physics Today* **56** (Dec. 2003), p. 56; see their reference 3.
- Bohr’s friend H.M. Hansen points out that hydrogen atoms absorb and emit light at specific wavelengths. Back in 1885, a Swiss high school math teacher named Johan Jacob Balmer had discovered a pattern to the hydrogen spectrum:

$$\frac{1}{\lambda} = \text{const.} \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

- Others then found similar patterns using not  $1/2^2$  but  $1/1^2$  (the Lyman series),  $1/3^2$  (the Paschen series),  $1/4^2$  (the Brackett series), and  $1/5^2$  (the Pfund series).

Hydrogen spectrum



## So what would Bohr predict?

- Consider a transition from an initial state  $n_i$  to a final state  $n_f$ :

$$(31) \quad \Delta E = \frac{hc}{\lambda} = |E_{n_i} - E_{n_f}| = -Z^2 E_0 \left| \frac{1}{n_i^2} - \frac{1}{n_f^2} \right|$$

from which we obtain (with  $Z = 1$ )

$$(32) \quad \frac{1}{\lambda} = \frac{E_0}{hc} \left| \frac{1}{n_i^2} - \frac{1}{n_f^2} \right|.$$

Can you imagine how Bohr felt when he found this agreed with the Balmer series?

- It may have been through matching the hydrogen spectrum that Bohr found that  $l = n\hbar$  rather than  $l = nh$ .