Some properties of the nucleus

- Atomic mass unit: defined to be 1/12$^{\text{th}}$ the mass of a ^{12}C atom.
  1 u=931.494 043 MeV/$c^2 = 1.66053886 \times 10^{-27}$ kg
- Proton: charge +1, mass 1.007 276 466 88 u or 938.272 029 MeV/c^2, spin $+\frac{1}{2}$.
- Neutron: charge 0, mass 1.008 664 915 60 u or 939.565 360 MeV/c^2, spin $+\frac{1}{2}$
- Nuclear density is approximately constant for all nuclei, so $\dfrac{A}{(4/3)\pi R^3} =$ constant or $R = R_0 A^{1/3}$ with $R_0 \simeq 1.2 \times 10^{-15}$ m.
- There are three types of decay emissions: alpha (α are $^4_2$He nuclei), beta (β are electrons and positrons), and gamma (γ: 0.1-10 MeV photons).
- Nuclear density is very high! Roughly 10^{17} kg/m^3

Stable isotopes

Masses of most isotopes are shown in Appendix B of Serway Stable and unstable isotopes (Serway Fig. 13.4; the plot shown here is Fig. 12.7 of Krane *Modern Physics* which swaps the Z and N axes):

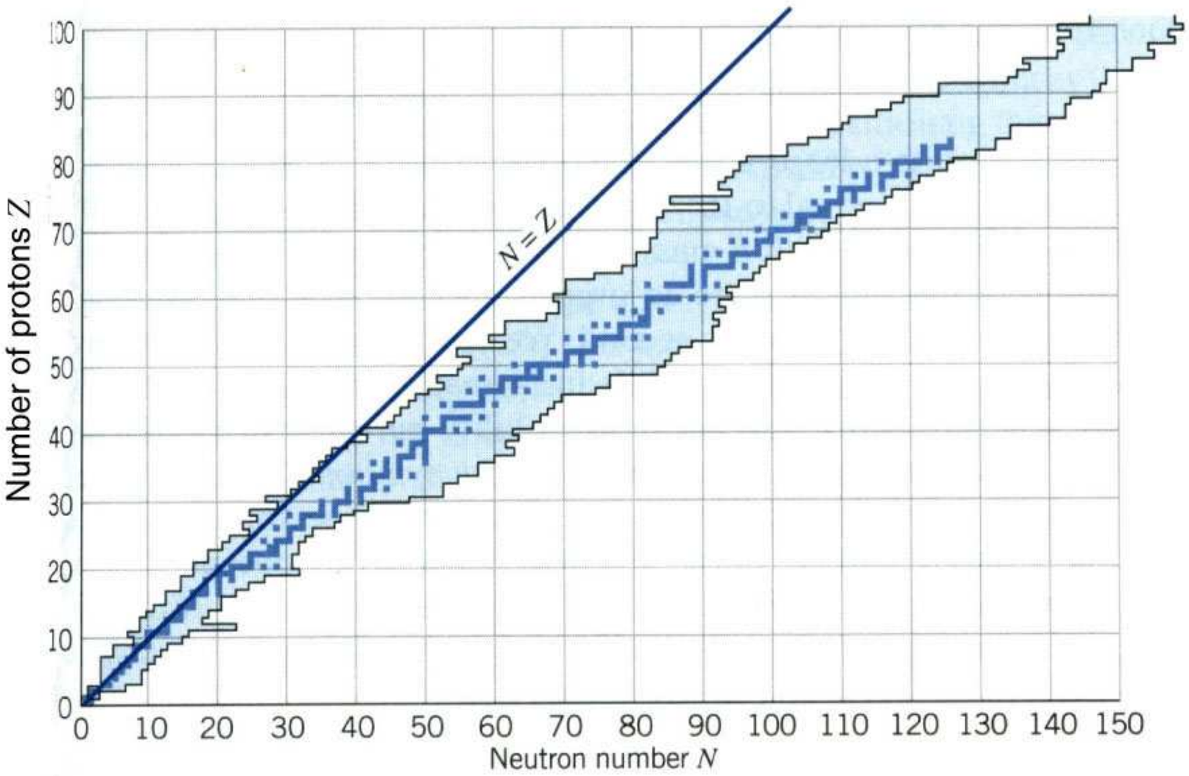

FIGURE 12.7 Stable nuclei are shown in dark; known radioactive nuclei are in light shading.

We will return to this in more detail in the liquid-drop model of the nucleus.

Radioactivity

- We cannot predict when any one nucleus will decay. We find, however, that the rate of nuclei that decay is proportional to the number present (Serway Eq. 13.8):

$$\frac{dN}{dt} = -\lambda N \tag{1}$$

- The activity $\mathcal{A} = \lambda N$ (or $R = \lambda N$ in Serway Eq. 13.10) is often expressed in Curies: 1 curie (Ci)=3.7×10^{10} decays/second.
- Integrating Eq. 1 gives $N = N_0 e^{-\lambda t}$ where N_0 is the number of nuclei present at $t = 0$. Since $\mathcal{A} \propto N$ we also have $\mathcal{A} = \mathcal{A}_0 e^{-\lambda t}$.
- It is frequently useful to talk about a half-life, which is the time over which half the nuclei decay:

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \qquad \rightarrow \qquad t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- Since $\mathcal{A} = \lambda N$, $\mathcal{A} \propto 1/t_{1/2}$. That is, a source with a shorter half-life is “hotter.” (The general public automatically assumes that long half-lives are worse).

A radioactive source

- Consider a 1 μCi source of ^{137}Cs:

$$^{137}_{55}\mathrm{Cs} \longrightarrow \beta^- + ^{137}_{56}\mathrm{Ba}$$

  Note how we have preserved mass (at least approximately) and charge.

- $t_{1/2} = 30.17$ years so

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{(30.17\ \mathrm{y}) \cdot (365\ \mathrm{d/y}) \cdot (24\ \mathrm{h/d}) \cdot (3600\ \mathrm{s/h})} = 7.23 \times 10^{-10}\ \mathrm{sec}^{-1}$$

- Number of atoms is

$$N = \frac{A}{\lambda} = \frac{(10^{-6}\ \mathrm{Ci}) \cdot (3.7 \times 10^{10}\ \mathrm{decays/s/Ci})}{7.23 \times 10^{-10}\ \mathrm{s}^{-1}} = 5.1 \times 10^{13}\ \mathrm{atoms}$$

- Mass is

$$\frac{5.1 \times 10^{13}\ \mathrm{atoms}}{6.02 \times 10^{23}\ \mathrm{atoms/mol}} \cdot 137\ \mathrm{g/mol} = 1.16 \times 10^{-8}\ \mathrm{g}$$

  though in fact we will see that $A = 137$ g/mol is not quite correct.

Alpha decays

The following table (Table 12.2 of Krane *Modern Physics*) is more informative than Table 13.5 of Serway:

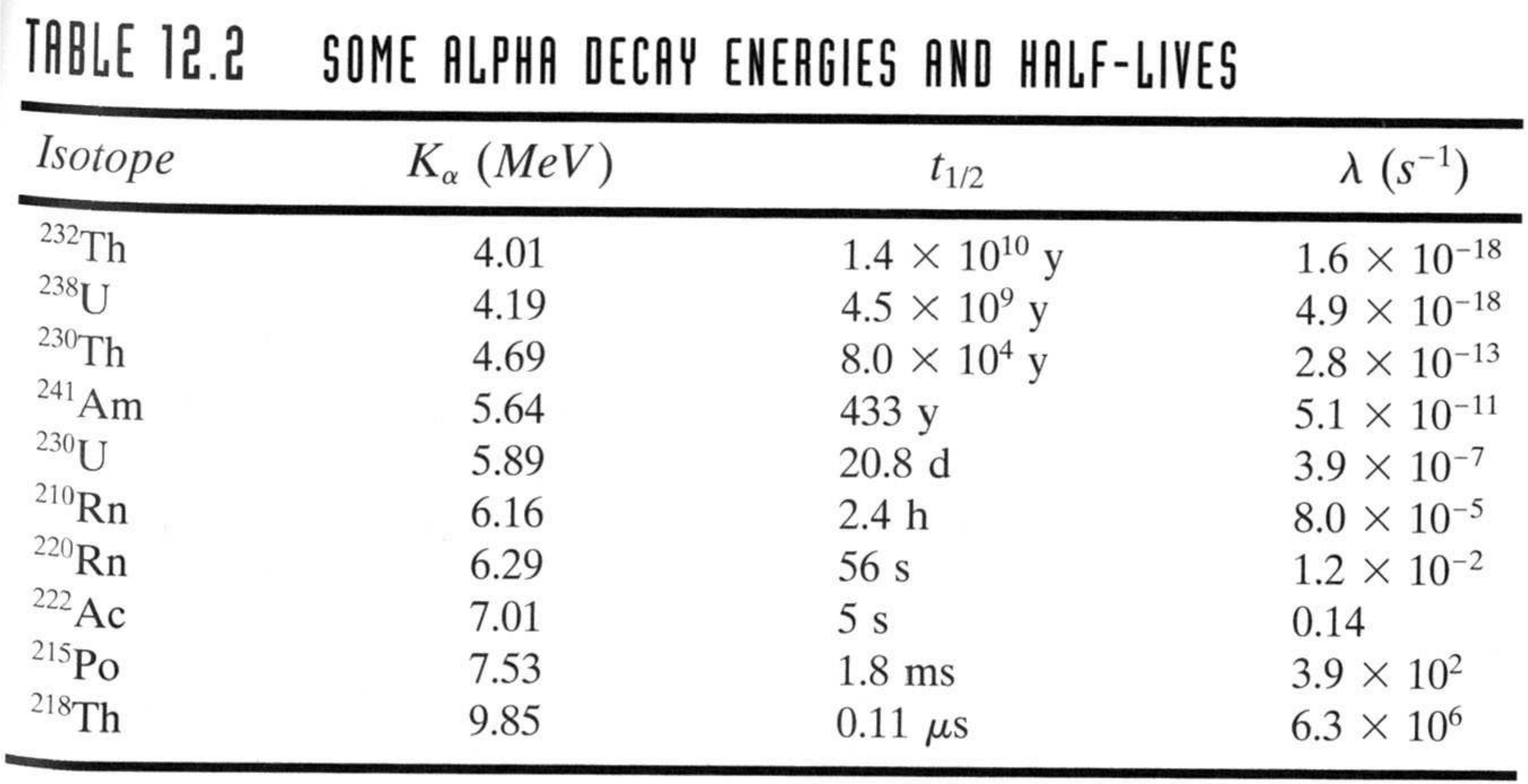

TABLE 12.2 SOME ALPHA DECAY ENERGIES AND HALF-LIVES

Isotope	K_α (*MeV*)	$t_{1/2}$	λ (s^{-1})
^{232}Th	4.01	1.4×10^{10} y	1.6×10^{-18}
^{238}U	4.19	4.5×10^{9} y	4.9×10^{-18}
^{230}Th	4.69	8.0×10^{4} y	2.8×10^{-13}
^{241}Am	5.64	433 y	5.1×10^{-11}
^{230}U	5.89	20.8 d	3.9×10^{-7}
^{210}Rn	6.16	2.4 h	8.0×10^{-5}
^{220}Rn	6.29	56 s	1.2×10^{-2}
^{222}Ac	7.01	5 s	0.14
^{215}Po	7.53	1.8 ms	3.9×10^{2}
^{218}Th	9.85	0.11 μs	6.3×10^{6}

Alpha decay: tunneling out of the nucleus

- George Gamow, 1928, while visiting Bohr in Copenhagen and Born and Heisenberg in Göttingen.
- There must be a strong but short-range nuclear force that overwhelms Coulomb repulsion.
- An alpha particle “trapped” in the nucleus will bang into the “walls” at time intervals of $2R/v$ or about 10^{22} times per second.
- Tunneling rate out of the nuclear potential goes like (Krane Eq. 12.24; see Serway Example 13.9 on p. 487)

$$\lambda = \frac{v}{2R} e^{-2kL} \text{ with } k = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

George Gamow (1904-1968)

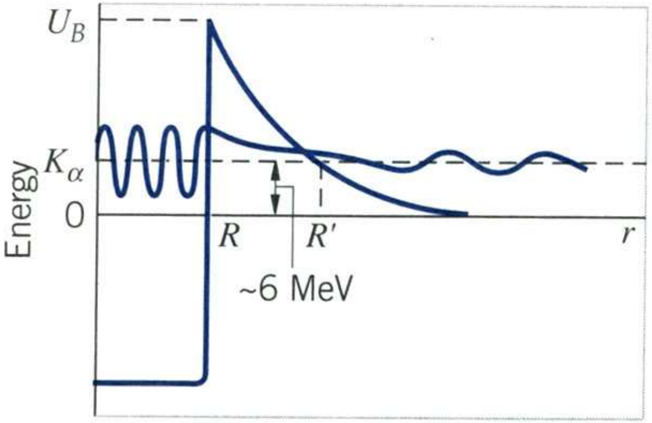

FIGURE 12.11 Barrier penetration by an alpha particle.

Alpha decay of radium

- The reaction is

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He} + \text{energy}$$

- Mass disappears!

$$\Delta m = (226.025402 - 222.017570 - 4.002603)\ \text{u} = 0.00523\ \text{u}$$

Einstein: $E = mc^2$ so ‘disappeared’ mass becomes energy Q (see Serway Eq. 13.16)

$$Q = (\Delta m)c^2 = 0.00523\ \text{u} \cdot \frac{931.5\ \text{MeV}/c^2}{\text{u}} = 4.87\ \text{MeV}$$

- That energy Q gets split between α and nucleus x:

$$Q = \frac{1}{2}\left[m_x v_x^2 + m_\alpha v_\alpha^2\right]$$

Alpha decay of radium II

- Conservation of momentum: $m_x v_x = m_\alpha v_\alpha$, so inserting this into $Q = (1/2)\left[m_x v_x^2 + m_\alpha v_\alpha^2\right]$ gives

$$Q = \frac{1}{2}\left[\frac{m_\alpha^2 v_\alpha^2}{m_x} + \frac{m_\alpha^2 v_\alpha^2}{m_\alpha}\right] = \frac{1}{2} m_\alpha v_\alpha^2 \left[\frac{m_\alpha}{m_x} + 1\right]$$

- Now $K_\alpha = (1/2) m_\alpha v_\alpha^2$ and $m_x \simeq m_\alpha \dfrac{A-4}{4}$ so $\dfrac{m_\alpha}{m_x} = \dfrac{4}{A-4}$
- Therefore

$$\left[\frac{m_\alpha}{m_x} + 1\right] = \left[\frac{4}{A-4} + \frac{A-4}{A-4}\right] = \frac{A}{A-4}$$

- We then have $Q = K_\alpha \dfrac{A}{A-4}$ or $K_\alpha = Q\dfrac{A-4}{A}$ (compare with Serway Eq. 13.19, which is written in terms of K_α and the mass of the daughter rather than the parent).
- In our example, $K_\alpha = 4.87 \text{ MeV} \dfrac{226-4}{226} = 4.79 \text{ MeV}$ so m_x carries little kinetic energy.

Beta decay

- Beta or β decay involves the emission of an electron (β^-) or its antimatter partner, a positron (β^+), from the nucleus.
- Remember that in alpha decay we found a well-defined relationship between the energy of the emitted alpha K_α and the total energy released Q of $K_\alpha = Q\frac{A-4}{A}$ (compare with Serway Eq. 13.19, which is written in terms of K_α and the mass of the daughter rather than the parent). This was due to requiring the simultaneous conservation of energy and momentum. Finding such well-defined energies of decays implied:
  - → Some set of discrete energy states in the nucleus
  - → The energy Q released is the difference between two states

Beta decay II

- Experimentally it is found that β decay yields a wide range of kinetic energies. If discrete energy states are producing the betas, how can we explain a distributed spectrum?
  - $\longrightarrow$ There must be a partner particle emitted in beta decay with which to share momentum and energy.
  - $\longrightarrow$ It must be electrically neutral.
  - $\longrightarrow$ From examining mass differences and energy release before and after decay, we can calculate $\Delta m\, c^2$ and infer that the particle must be very light.
- In 1930, Wolfgang Pauli proposes the existence of the neutrino ν. No charge, almost no mass (present experimental limits are roughly 0.1-2 eV/c^2 for the electron neutrino)
- In fact, there are two other leptons besides the electron (μ at 105 MeV/c^2, τ at 1777 MeV/c^2), and each lepton has its own neutrino type ν_μ and ν_τ. They are rare and we won't discuss them further.
- The force that describes this is the *weak* nuclear force (and at high energies we have electroweak unification).

Example beta decays

- Decay of the neutron produces an electron and an *anti*-neutrino:

  $n \rightarrow p + e^- + \bar{\nu}$ or in nuclei ${}^A_Z X_N \rightarrow {}^A_{Z+1} X'_{N-1} + e^- + \bar{\nu}$

  Energy released: $Q = [m(^A X) - m(^A X')]c^2$ (after keeping track of the electron gained by the ${}_{Z+1}X'$ nucleus)

- Decay of the proton (lifetime of free protons is at least 10^{33} years):

  $p \rightarrow n + e^+ + \nu$ or in nuclei ${}^A_Z X_N \rightarrow {}^A_{Z-1} X'_{N+1} + e^+ + \nu$

  Energy released: $Q = [m(^A X) - m(^A X') - 2m_e]c^2$

- Another reaction is *electron capture* by a proton:

  $p + e^- \rightarrow n + \nu$ or in nuclei ${}^A_Z X_N + e^- \rightarrow {}^A_{Z-1} X'_{N+1} + \nu$

  Energy released: $Q = [m(^A X) - m(^A X')]e^2$ given almost entirely to the neutrino.

See Serway Table 13.4, or this table:

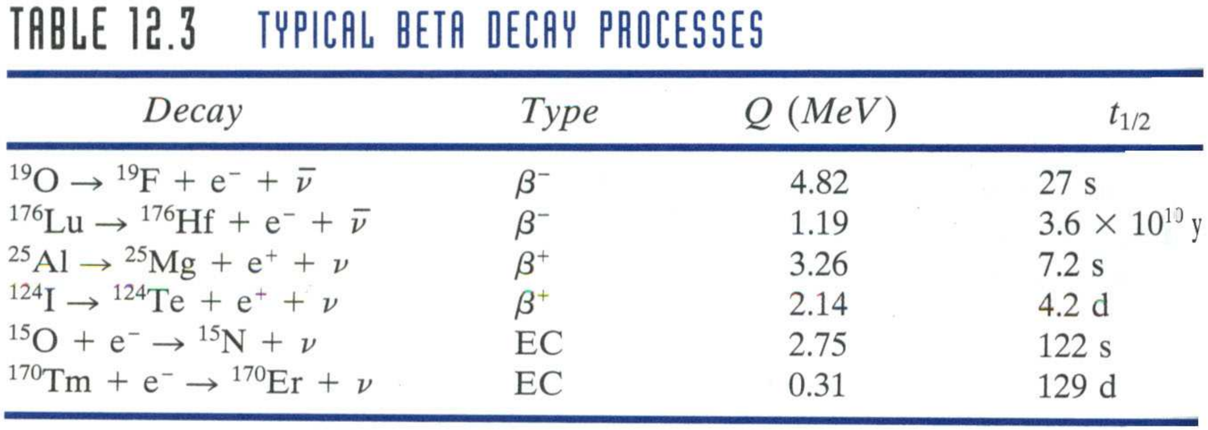

TABLE 12.3 TYPICAL BETA DECAY PROCESSES

Decay	*Type*	*Q (MeV)*	$t_{1/2}$
${}^{19}O \rightarrow {}^{19}F + e^- + \bar{\nu}$	β^-	4.82	27 s
${}^{176}Lu \rightarrow {}^{176}Hf + e^- + \bar{\nu}$	β^-	1.19	3.6×10^{10} y
${}^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
${}^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
${}^{15}O + e^- \rightarrow {}^{15}N + \nu$	EC	2.75	122 s
${}^{170}Tm + e^- \rightarrow {}^{170}Er + \nu$	EC	0.31	129 d

Gamma decay

Photons released as a result of transitions between nuclear energy levels (just like with electronic transitions, except the energies are much larger!). See Serway Fig. 13.20 for one example; here's another:

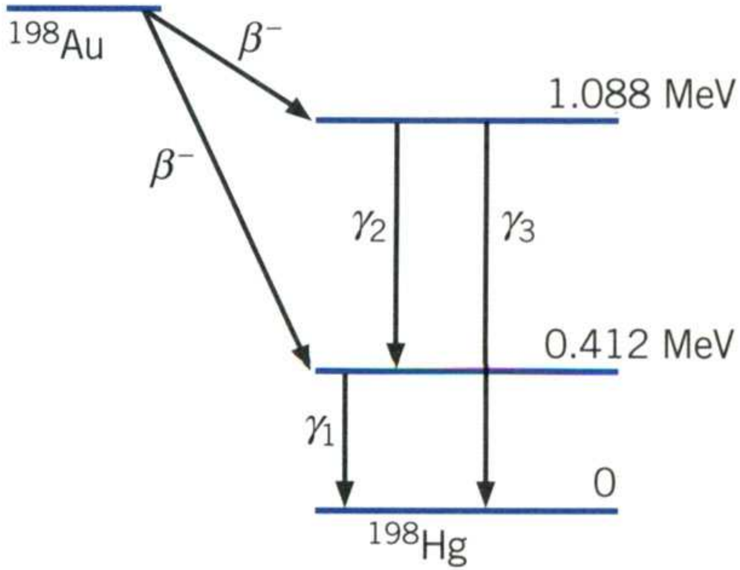

FIGURE 12.15 Some gamma rays emitted following beta decay.

Decay chains

We've outline the basic radioactive decay processes. In fact, radioactive isotopes often involve a series of decays, and they can include branching:

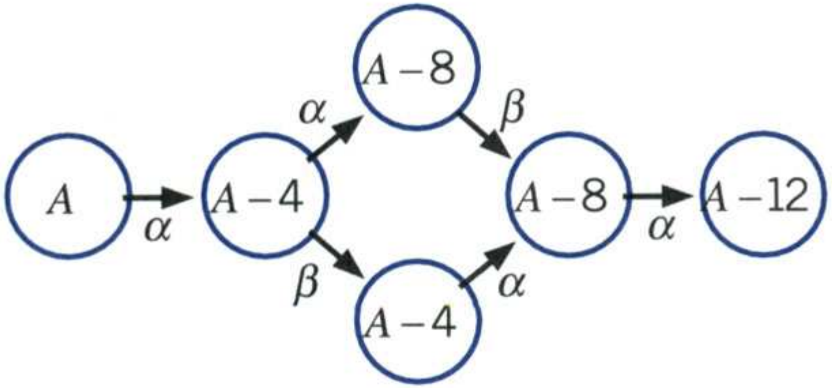

FIGURE 12.16 An example of a hypothetical radioactive decay chain.

Three natural heavy-element decay chains wind up producing different isotopes of lead:

${}^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}$ Pb ($t_{1/2} = 4.47 \times 10^9$ years)

${}^{235}_{92}\mathrm{U} \rightarrow {}^{207}_{82}$ Pb ($t_{1/2} = 7.04 \times 10^8$ years)

${}^{232}_{90}\mathrm{Th} \rightarrow {}^{208}_{82}$ Pb ($t_{1/2} = 1.41 \times 10^9$ years)

An example decay chain

The half-life of ${}^{235}_{92}\text{U} \rightarrow {}^{207}_{82}$ Pb is $t_{1/2} = 7.04 \times 10^8$ years

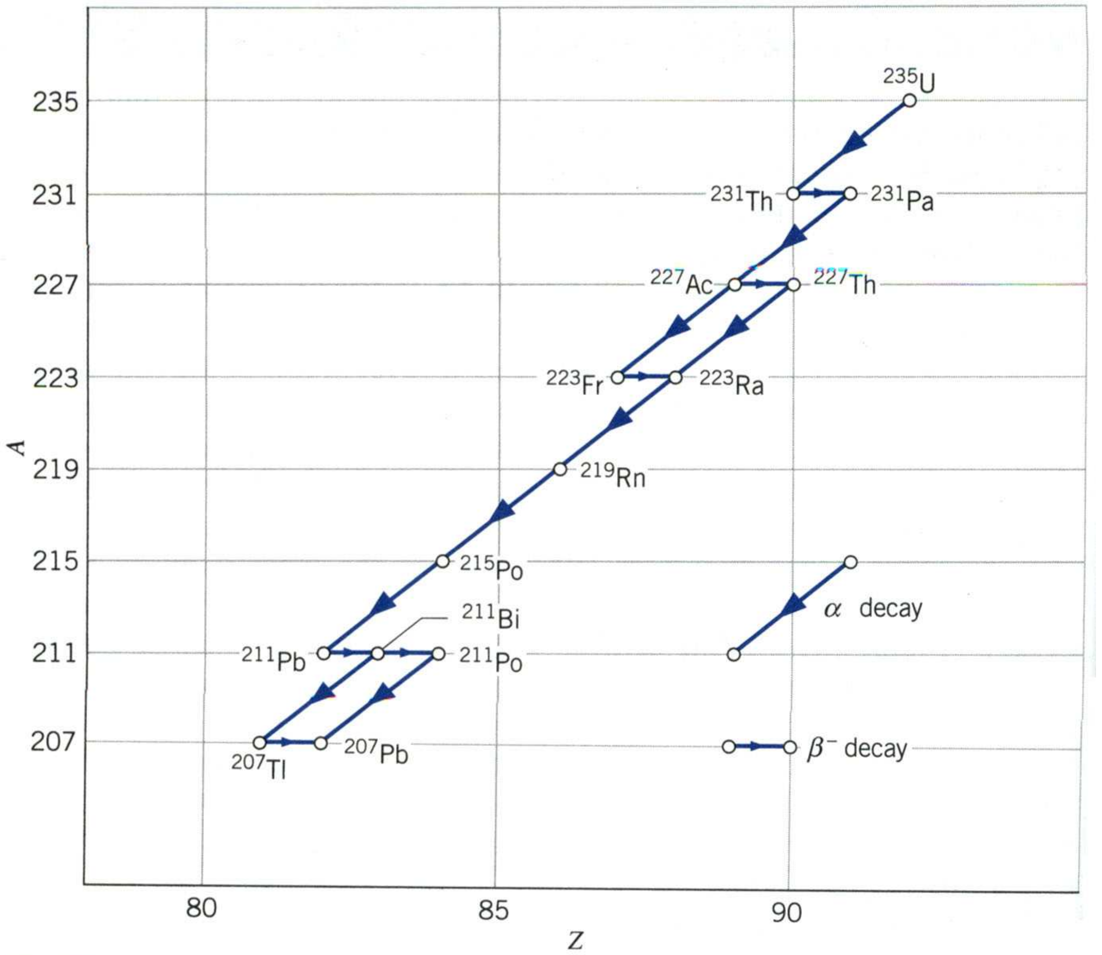

FIGURE 12.17 The ^{235}U decay chain.

See also Serway Fig. 13.21.

Stable isotopes (again)

Why are only certain steps allowed on the chain? We will soon do the liquid-drop model of the nucleus.

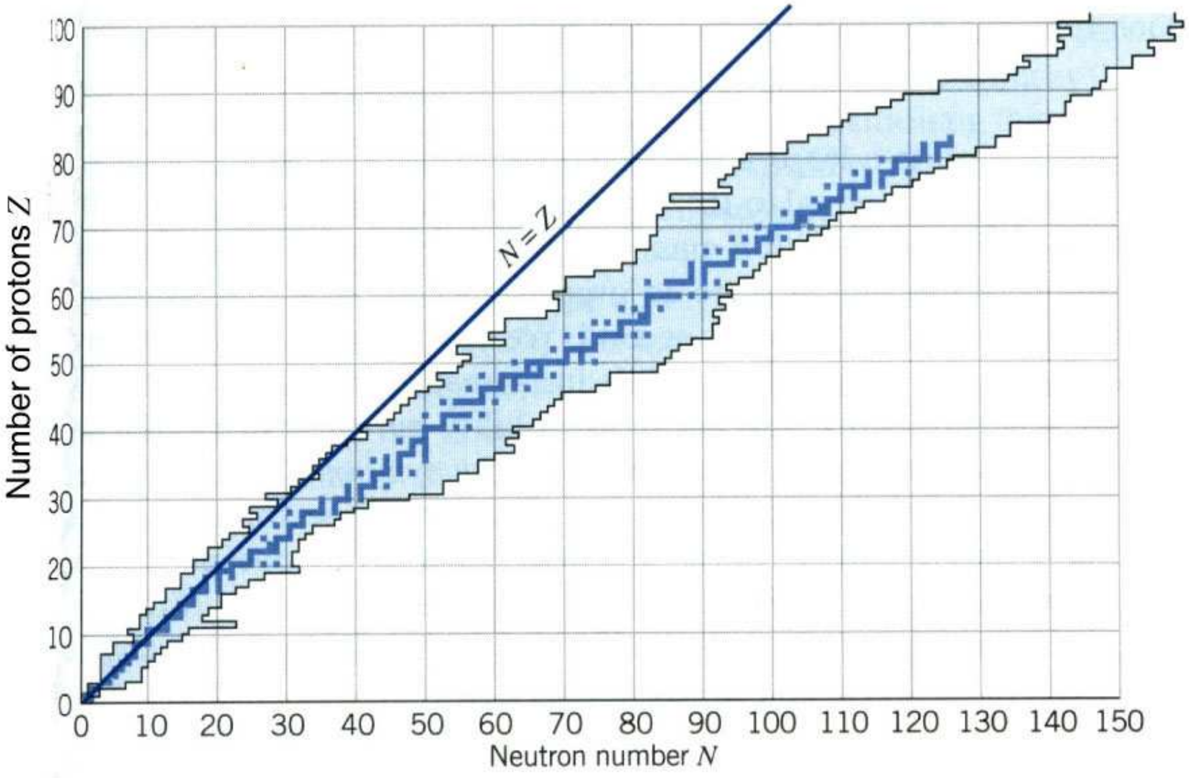

FIGURE 12.7 Stable nuclei are shown in dark; known radioactive nuclei are in light shading.

Radioactive dating

- If we assume that we start out with all parent nuclei, we can measure the ratio of daughter to parent nucleii (for example by using a mass spectrometer) to determine the age of an object.
- Consider the case of carbon dating. ^{14}C is created at a steady rate from ^{12}C by cosmic ray irradiation of the atmosphere, leading to a steady ratio ^{14}C/^{12}C=1.3×10^{-12}.
- Note that carbon content of the atmosphere is increasing: CO_2 has gone from 280 ppmv in the year 1800, to 378 ppmv at the end of the year 2004. Methane or CH_4 is also increasing in the atmosphere although it is present in the atmosphere at only about 1.8 ppmv. However, the ^{14}C/^{12}C ratio should remain the same.
- Carbon in the atmosphere is taken up by plants which are then eaten by animals, so that everything gets about the same ^{14}C/^{12}C ratio. When something (you, a tree, or a rabbit) dies, it stops cycling carbon.
- We can measure the ^{14}C/^{12}C ratio in a mass spectrometer, but only if we can break apart all molecules (for example, rip the carbons out of the amino acid glycine $C_2H_5NO_2$).

An example of using carbon dating

- Example 13.11, Serway p. 490. To repeat our basic information: $t_{1/2}$ for ^{14}C is 5730 years, or

$$\lambda = \frac{\log(2)}{t_{1/2}} = \frac{0.693}{5730\text{ y}}\,\frac{1\text{ y}}{365\text{ d}}\,\frac{1\text{ d}}{24\text{ h}}\,\frac{1\text{ h}}{3600\text{ s}} = 3.83\times10^{-12}\text{ s}^{-1},$$

  and “fresh” carbon has the ratio ^{14}C/^{12}C=1.3×10^{-12}.

- The intial number of ^{12}C atoms is

$$\frac{25.0\text{ g}}{12.0\text{ g/mol}}\,\frac{6.02\times10^{23}\text{ atoms}}{1\text{ mol}} = 1.26\times10^{24}\text{ atoms}$$

  The initial number of ^{14}C atoms is $1.3\times10^{-12}\cdot1.26\times10^{24} = 1.6\times10^{12}$ atoms.

- The activity of 25.0 g of “fresh” carbon due to its ^{14}C mass fraction is thus

$$R_0 = N_0\lambda = (1.6\times10^{12}\text{ atoms})\cdot(3.83\times10^{-12}\text{ s}^{-1}) = 6.13\text{ decays/s}$$

  or 370 decays/minute.

Carbon dating II

- Let's say we measure an activity in "old" carbon of 250 decays/minute. We have

$$\frac{R}{R_0} = \frac{N\lambda}{N_0\lambda} = e^{-\lambda t}$$

because $N = N_0 e^{-\lambda t}$. Taking the logarithm of both sides gives

$$\begin{aligned} \ln(R/R_0) &= -\lambda t = \frac{\ln 2}{t_{1/2}}\, t \\ t &= t_{1/2}\frac{\ln(R_0/R)}{\ln 2} = (5730\text{ y})\frac{\ln(370/250)}{\ln 2} = 3240\text{ y} \end{aligned}$$

- Note that when $R \to 0$ we have problems; it's hard to date back more than about 10 half-lives.