The setting for Modern Physics

Physics had reached a point of considerable triumph by the year 1900: we had Newtonian mechanics to explain the motion of the planets, thermodynamics and statistical mechanics to understand heat engines and their efficiency, and a unified theory of electromagnetism. One could be smug!

> The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. . . Our future discoveries must be looked for in the sixth place of decimals. — *Attributed to Albert A. Michelson, 1894*

and

> There is nothing new to be discovered in physics now. All that remains is more and more precise measurement. — *Attributed to William Thomson, 1st Baron Kelvin, in an address to the British Association for the Advancement of Science, 1900*

An aside: getting the quotes right!

Here again is the Michelson quote in the form you will often find it:

> The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. . . Our future discoveries must be looked for in the sixth place of decimals.

The correct quote seems to be as follows:

> It is never safe to affirm that the future of physical science has no marvels in store which may be even more astonishing than those of the past; but it seems probable that most of the grand underlying principles have now been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice.

followed by a statement several hundred words later of

> An eminent physicist has remarked that the future truths of physical science are to be looked for in the sixth place of decimals.

Yogi Berra: “I really didn’t say everything I said”

- These two versions of Michelson’s words are a bit different! The second [correct] one adds “It is never safe to affirm that the future of physical science has no marvels in store. . .”
- The first version of the quote is the one you’ll commonly find if you do a Google search. As an example, `http://amasci.com/weird/end.html` states that Michelson said this in a speech at the dedication of the Ryerson Physics Lab at the University of Chicago in 1894.
- According to David Henige, “Mis/Adventures in Mis/Quoting,” *Journal of Scholarly Publishing* **32** (3), 123–135 (2001) (also reproduced at `http://128.100.205.52/product/jsp/323/quoting1.html`), the second version of the quote is the earliest one he was able to find from original source material: Albert A. Michelson, “Some of the Objects and Methods of Physical Science,” *University of Chicago Quarterly Calendar* **10**, 15 (August 1894).
- So: if you *really* want to get a quote right, track it back to the earliest possible source. If that’s not feasible, then at least say something like “Quote attributed to Michelson by [source].”
- The internet is handy, but you can’t trust the accuracy of everything you find on the internet!

Back to physics at the turn of the previous century

- The theme of this course is that the physics you have learned in your freshman courses, which is mainly “classical physics” that was largely worked out before 1900, is incomplete.
- It’s not wrong; it works with remarkable accuracy in nearly all situations.
- It’s just incomplete, in that it does not full describe the very small, the very fast, and the very energetic.
- As we explore the situations where classical physics breaks down, keep in mind that we will usually be able to apply a *correspondance principle* and show that the more correct modern physics theory reproduces the classical physics theory in less extreme circumstances.

Maxwell's equations of electricity and magnetism

Maxwell unified some laws in 1855; by 1873 he found that all of these laws could all be represented by four partial differential equations. A triumph of unification!

$$\begin{aligned}
\text{Faraday:} \quad \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\text{Ampère:} \quad \vec{\nabla} \times \vec{B} &= \mu(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}) \\
\text{Gauss (electric):} \quad \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon} \\
\text{Gauss (magnetic):} \quad \vec{\nabla} \cdot \vec{B} &= 0.
\end{aligned}$$

James Clerk Maxwell (1831–1879)

MAT 203/205: $\vec{\nabla}\times$ is called the *curl* (vector cross-product), and $\vec{\nabla}\cdot$ is called the *divergence*. Using Ohm's law $\vec{J} = \sigma \vec{E}$, Ampère's law becomes

$$\vec{\nabla} \times \vec{B} = \sigma\mu\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}.$$

Electromagnetic wave propagation

You'll see this in PHY 301: can manipulate Maxwell's equations to give

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \text{and} \qquad \nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

Wave solutions for electric field:

$$\text{Plane: } \vec{E} = \text{Re}\left[\vec{E}_0 e^{-i(\vec{k}\cdot\vec{z}-\omega t)}\right] \qquad \text{Spherical: } E = \text{Re}\left[E_0 \frac{e^{-i(kr-\omega t)}}{r}\right].$$

Velocity of wave propagation:

$$c \equiv \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \text{ m/sec}$$

or 30 cm per nanosecond.

Speed of light

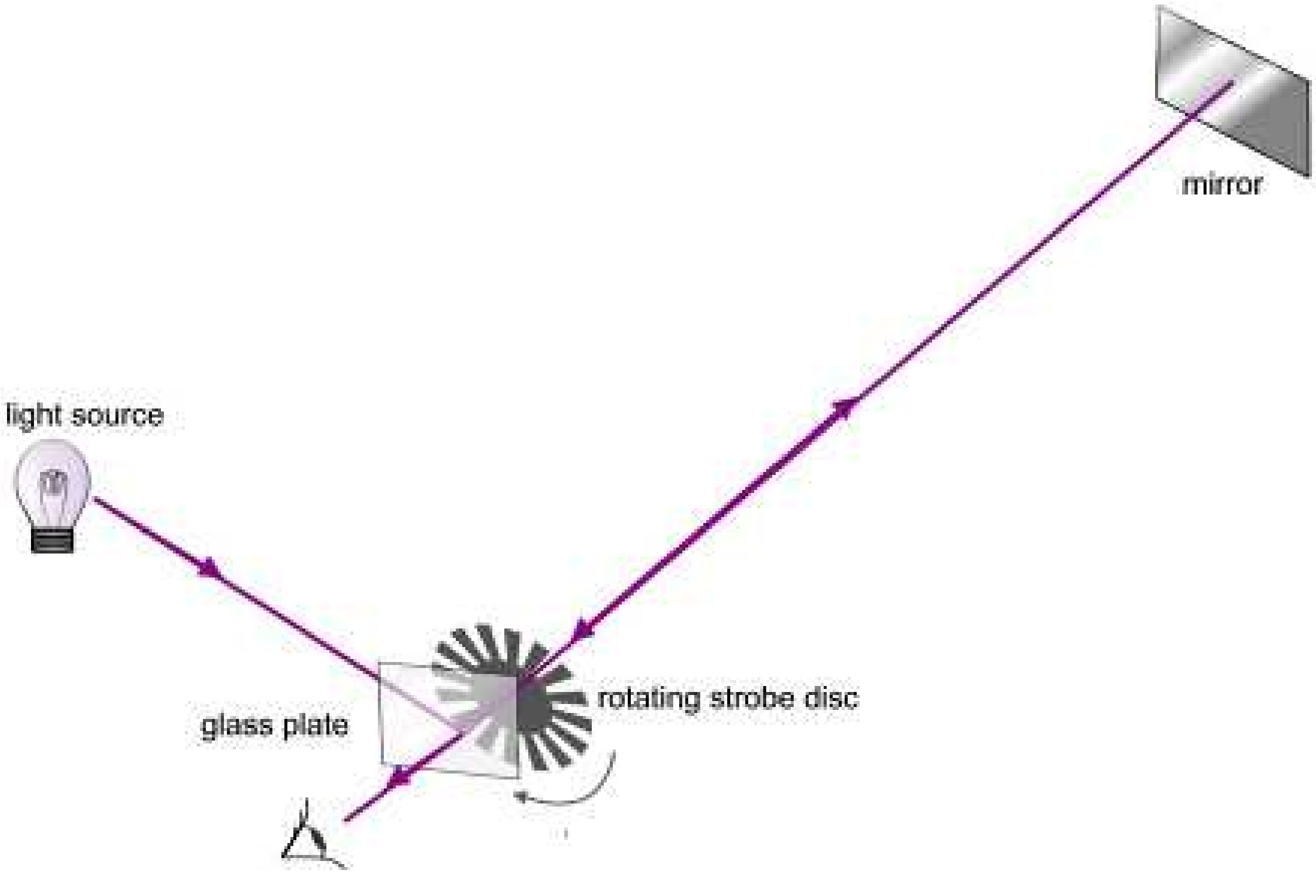
- Inferred to be finite from astronomical measurements by Romer (1676)
- Huygens soon estimated 2.2×10^8 m/sec.
- Refined by Bradley (1728) to 2.98×10^8 m/sec.
- First terrestrial measurement by Fizeau (1849): 3.13×10^8 m/sec.

Maxwell on the similarity of $1/\sqrt{\mu_0 \epsilon_0}$ to the above, as quoted in Griffiths' *Introduction to Electrodynamics* (Prentice-Hall, 1981):

> *This velocity is so nearly that of light, that it seems we have strong reasons to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.*

Armand Hippolyte Louis Fizeau, 1819–1896

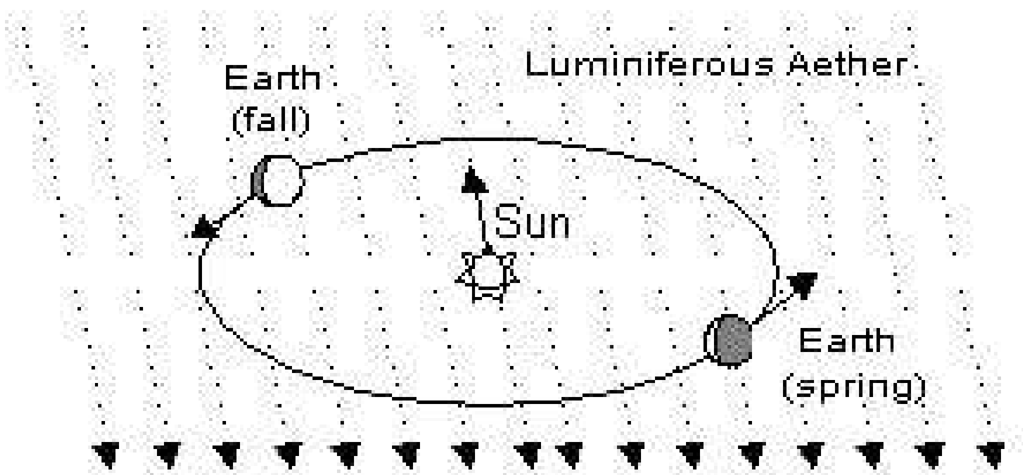
Fizeau's experiment



As drawn on Wikipedia by Theresa Knott.

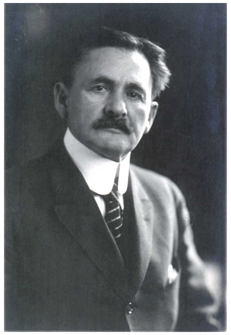
Speed of medium?

- Sound waves travel at speed of sound $v = \sqrt{(c_p/c_v)(P/\rho)}$ relative to speed of medium (wind speed).
- What's the medium for light? The æther. Speed of earth in its orbit: 3×10^4 m/sec (see homework).
- Differences in light speed?
- How does that fit into $c = 1/\sqrt{\mu_0 \epsilon_0}$?

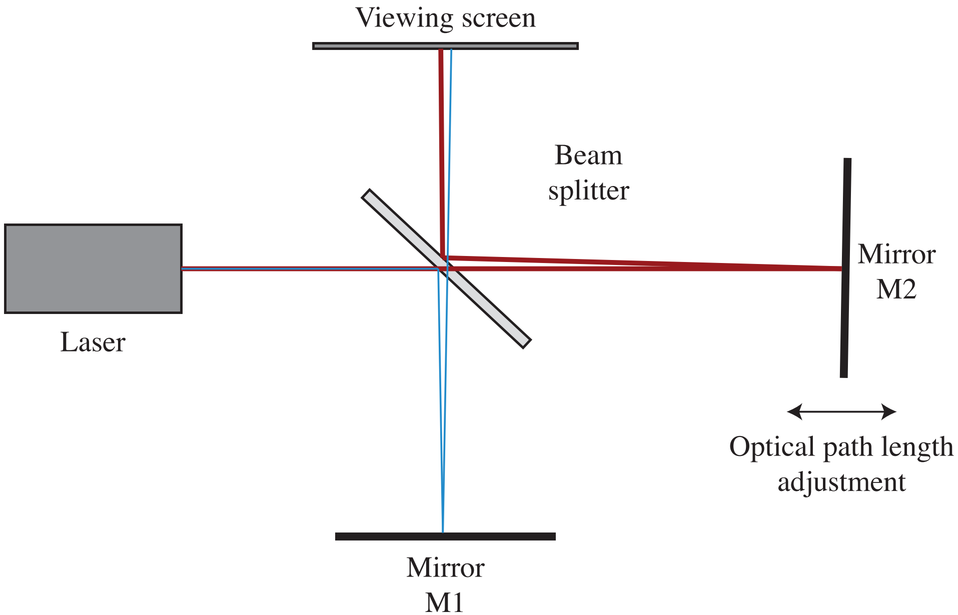


Michelson and Morley

- Albert Michelson (Case-Western, 1881): interferometer. Fringe shift of 0.02; expected 0.04.
- Joined by Edward Morley in attempts to measure æther wind with 11 m long interferometer.
- Improved result (1887): fringe shift < 0.01 when 0.4 expected, so $v_{\text{æther}} < 8 \times 10^4$ m/sec.
- Nobel Prize 1907 (Michelson)



Albert A. Michelson (1852–1931)



Classical relativity

Positions in inertial frame 2 versus frame 1:

$$
\begin{aligned}
x_2 &= x_1 - vt \\
y_2 &= y_1 \\
z_2 &= z_1 \\
t_2 &= t_1
\end{aligned}
$$

Velocity in inertial frame 2 versus frame 1:

$$
\begin{aligned}
v_{2x} &= v_{1x} - v \\
v_{2y} &= v_{1y} \\
v_{2z} &= v_{1z}
\end{aligned}
$$

So if you're in a spaceship traveling at half the speed of light, and you turn on your headlights, what's the velocity of the light coming out of your headlights?

Measuring a speedy meter stick

Meter stick traveling at velocity $-v$ relative to observer. Mark the positions of the two ends A and B:

$$
\begin{aligned}
A_2 &= A_1 + vt \\
B_2 &= B_1 + vt \\
B_2 - A_2 &= B_1 + vt - (A_1 + vt) = B_1 - A_1
\end{aligned}
$$

So from this simple addition of object and light velocity vectors, the length you observe for the meter stick does not change with its travel velocity.

Enter Einstein

Before we look at how Einstein thought about this problem, let's start with learning who he was. It is widely reported that Einstein was a learning disabled student. Here's a quote from `http://www.nald.ca/ldanl/famous.htm` which claimed that Einstein was a learning disabled student:

> Albert Einstein did not speak until the age of three. Even as an adult Einstein found that searching for words was laborious. He found school work, especially math, difficult and was unable to express himself in written language. He was thought to be simple minded (retarded), until it was realized that he was able to achieve by visualizing rather than by the use of language. His work on relativity, which revolutionized modern physics, was created in his spare time.

Again, don't trust everything you read on the internet! A short response to this is given on another web page `http://www.audiblox2000.com/dyslexia_dyslexic/dyslexia005.htm` based on two complete biographies with citations to their source material and all that.

Einstein's early career

- Finished PhD 1901.
- Working in Swiss Patent Office in Bern while completing *privat-dozent* (called the Habilitation today in Germany; it's almost a sort of second PhD to prove that you're enough of a scholar to be a professor) so that he could seek a university professor position.
- 1905: what a year! Read Abraham Pais' book "Subtle is the Lord" to learn more.
- 1908: Received his *Privatdozent* at Bern
- 1911: Professor at the University of Prague
- 1914: Professor at the University of Berlin
- 1921: Nobel Prize for photoelectric effect.

Einstein's annus mirablis*: 1905*

Four papers to *Annalen der Physik*:

- "On the Motion - Required by the Molecular Kinetic Theory of Heat - of Small Particles Suspended in a Stationary Liquid." Direct evidence of the existence of atoms.
- "On a Heuristic Viewpoint Concerning the Production and Transformation of Light," interpreting observations of the photoelectric effect as indicating the existence of photons. Nobel Prize, 1921.
- "On the Electrodynamics of Moving Bodies," concerning what is now called special relativity.
- "Does the Inertia of a Body Depend Upon Its Energy Content?," containing $E = mc^2$.

Quite a year!

Albert Einstein (1879–1955) in 1905.

Photo source

Einstein's 3$^{\text{rd}}$ paper: motivation

First page of Einstein's 1905 paper "On the Electrodynamics of Moving Bodies" (translation quoted From Griffiths, *Introduction to Electrodynamics*, Prentice-Hall, 1981, p. 392):

> It is known that Maxwell's electrodynamics–as usually understood at the present time–when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. . .
>
> Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relative to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest.

Einstein's postulates

Special relativity: two frames with velocity difference (general relativity deals with large acceleration differences). Postulates:

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light in free space has the same value $c = 1/\sqrt{\mu_0 \epsilon_0}$ in all inertial reference frames.

Postulate 2 explains Michelson-Morley. But what about the speed of light coming out from the headlights of a spaceship?

Light pulse from a point

At $t_1 = t_2 = 0$, a spherical wave light pulse is emitted from a common source point. Frame S_2 is moving at a speed v in the $\hat{x}$ direction relative to frame S_1. Positions of pulse at a later time:

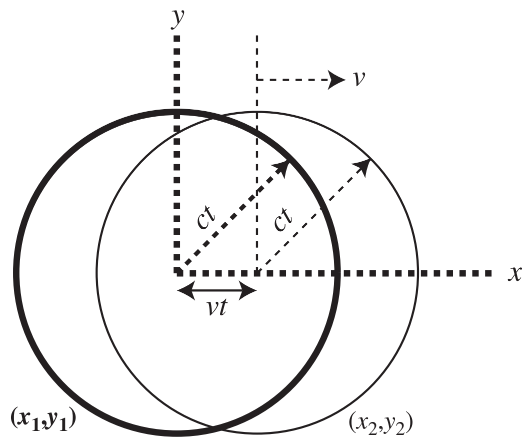
$$(1) \qquad x_1^2 + y_1^2 + z_1^2 - c^2 t_1^2 = 0$$

$$(2) \qquad x_2^2 + y_2^2 + z_2^2 - c^2 t_2^2 = 0.$$

Applying Galilean relativity

$$x_2 = x_1 - vt_1$$
$$y_1 = y_2$$
$$z_1 = z_2$$
$$(3) \qquad t_1 = t_2$$

gives inconsistencies! (Homework)



Observers in both frames see light pulse as a sphere expanding from an origin at the same velocity

How to fix it?

Motion is in $\hat{x}$, so assume $y_1 = y_2$ and $z_1 = z_2$ as before. Simplest fix is a linear correction factor γ:

(4) $$x_2 = \gamma(x_1 - vt_1)$$

Rewrite Eq. 2 and subtract y, z terms from Eqs. 1 and 2:

(5) $$x_1^2 - c^2t_1^2 = 0 = \gamma^2(x_1 - vt_1)^2 - c^2t_2^2 = \gamma^2(x_1^2 - 2x_1vt_1 + v^2t_1^2) - c^2t_2^2$$

Gives

(6) $$(1 - \gamma^2)x_1^2 + 2\gamma^2vx_1t_1 - \gamma^2v^2t_1^2 + c^2(t_2^2 - t_1^2) = 0.$$

It is inescapable that t_2 will depend on both t_1 and x_1.

Finding the fix

Because t_2 must depend on both t_1 and x_1, assume a solution for t_2 of the form

(7) $$t_2 = At_1 - Bvx_1.$$

Substituting this into Eq. 6 and collecting terms, we obtain

(8) $$(1 - \gamma^2 + c^2v^2B^2)x_1^2 + 2(\gamma^2 v - c^2vAB)x_1t_1 + (c^2A^2 - c^2 - \gamma^2v^2)t_1^2 = 0.$$

Quadratic equation, which would have only two solutions of x_1. But we need something that works for all x_1! Must make coefficients be zero:

(9) $$(1 - \gamma^2 + c^2v^2B^2) = 0 \quad \text{or} \quad c^2B^2 = \frac{\gamma^2 - 1}{v^2}$$

(10) $$(\gamma^2 v - c^2vAB) = 0 \quad \text{or} \quad \gamma^4 = c^2A^2\,c^2B^2$$

(11) $$(c^2A^2 - c^2 - \gamma^2v^2) = 0 \quad \text{or} \quad c^2A^2 = \gamma^2v^2 + c^2.$$

Simplifying

Again, we have $c^2 B^2 = (\gamma^2 - 1)/v^2$ from Eq. 9, and $c^2 A^2 = \gamma^2 v^2 + c^2$ from Eq. 11, and $c^2 A^2\, c^2 B^2 = \gamma^4$ from Eq. . We can then substitute Eqs. 9 and 11 into Eq. 21 to get (using $\beta \equiv v/c$)

$$\begin{aligned}
\gamma^4 &= c^2 A^2\, c^2 B^2 = (\gamma^2 v^2 + c^2)\frac{(\gamma^2 - 1)}{v^2} \\
&= (\gamma^2 + \frac{1}{\beta^2})(\gamma^2 - 1) = \gamma^4 + \frac{\gamma^2}{\beta^2} - \gamma^2 - \frac{1}{\beta^2} \\
\gamma^2(\frac{1}{\beta^2} - 1) &= \frac{1}{\beta^2} \\
\gamma^2 &= \frac{1}{\beta^2}\,\frac{1}{1/\beta^2 - 1} = \frac{1}{1 - \beta^2}
\end{aligned}$$

which finally gives

(12)
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}},$$

where $\beta \equiv v/c$.

Simplifying II

Let's turn back to Eq. 9:

$$\begin{aligned} c^2B^2 &= \frac{\gamma^2-1}{v^2} \\ c^4\beta^2B^2 &= \gamma^2-1 = \frac{1}{1-\beta^2}-1 = \frac{1}{1-\beta^2}-\frac{1-\beta^2}{1-\beta^2} = \frac{\beta^2}{1-\beta^2} = \beta^2\gamma^2 \\ c^4B^2 &= \gamma^2 \\ B &= \frac{\gamma}{c^2} \end{aligned} \tag{13}$$

Let's also turn back to Eq. 11 of $c^2A^2 = \gamma^2v^2 + c^2$:

$$A^2 = \gamma^2\beta^2 + 1 = \frac{\beta^2}{1-\beta^2} + \frac{1-\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2 \tag{14}$$

which gives $A = \gamma$ so that we can write our velocity transformation from Eq. 7 as

$$t_2 = At_1 - Bvx_1 = \gamma(t_1 - \frac{\beta}{c}x_1). \tag{15}$$

Lorentz transformations

We have shown that the net transformation between coordinate systems is

(16) $$x_2 = \gamma\,(x_1 - vt_1)$$

(17) $$y_2 = y_1$$

(18) $$z_2 = z_1$$

(19) $$t_2 = \gamma\left(t_1 - \frac{\beta}{c}x_1\right)$$

with γ found from Eq. 12 to be

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}},$$

This transformation was first worked out by H. Lorentz in 1904, but in the assumption of a moving æther; as a result, Einstein gets the credit for getting the physics right. Important mathematical contributions later added by Poincaré and Minkowski.

Lorentz factor γ

In the limit of $v \ll c$ or $\beta \ll 1$, we have

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = (1-\beta^2)^{-1/2} \simeq 1 + \frac{1}{2}\beta^2. \tag{20}$$

In the limit $\beta \to 0$, we have $\gamma \to 1$ and Eqs. 16 and 19 both revert to the Galilean results. This illustrates the all-important **correspondence principle** with classical physics.