PHY 251, Spring 2007: Equations for Final Exam

This is the January 30, 2007 version.

$p = h/\lambda$, $E = hc/\lambda = h\nu$ with $hc = 1240 \times 10^{-9}$ eV·m. 1 eV=1.602×10^{-19} J.

$h = 4.14 \times 10^{-15}$ eV·sec, $h = 6.63 \times 10^{-34}$ J·sec and $c = 3.00 \times 10^8$ m/sec.

Masses: $m_e = 510.998$ keV/c^2, $m_p = 938.272$ MeV/c^2, $m_n = 939.565$ MeV/c^2, 1 u=931.494 MeV/c^2. $\Delta E\,\Delta t \geq \hbar$, $\Delta x\,\Delta p \geq \hbar$.

Bohr model: $r_n = \dfrac{n^2}{Z} a_0$ with $a_0 = \dfrac{\epsilon_0 h^2}{m\pi e^2} = 0.053$ nm, and $E_n = -\dfrac{Z^2}{n^2} E_0$ with $E_0 = \dfrac{me^4}{8\epsilon_0^2 h^2} = 13.60$ eV.

$m_r = \dfrac{m_1 m_2}{m_1 + m_2}$. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2} = m\dfrac{v^2}{r}$. $-\dfrac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\dfrac{\partial\psi}{\partial t} = E\psi$ if time independent.

3D Schrödinger equation with Coulomb potential: $\psi(r, \theta, \varphi) = R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\varphi)$.

Volume element: $r^2 dr\, \sin\theta d\theta\, d\varphi$.

Infinite box: $\psi_n = \sqrt{2/L}\sin(n\pi x/L)$, with $E_n = n^2h^2/(8mL^2)$.

For V constant:

$E > V$: $A\sin kx + B\cos kx,\ k = \sqrt{2m(E - V)}/\hbar$, and

$E < V$: $C\exp[-kx],\ k = \sqrt{2m(V - E)}/\hbar$.

Harmonic oscillator: $E_n = (n + \frac{1}{2})\hbar\omega$, $\psi_0 = A\exp[-\omega m x^2/(2\hbar)]$.

Rotational states: $E = \dfrac{\hbar^2 \ell(\ell + 1)}{2m_r r_0^2}$.

Energy order of shells: $1s < 2s < 2p < 3s < 3p < 4s \lesssim 3d < 4p < 5s < 4d < 5p$
$< 6s < 4f \lesssim 5d < 6p < 7s < 6d \lesssim 5f \ldots$

$|L| = \sqrt{\ell(\ell + 1)}\hbar$, $L_z = m_\ell \hbar$, $\vec{\mu}_L = -(e/2m)\vec{L}$.

$|S| = \sqrt{s(s + 1)}\hbar$, $S_z = m_s\hbar$, $\vec{\mu}_s = -(e/m)\vec{S}$.

$U = m_\ell \mu_B B = 2m_s\mu_B B$ with $\mu_B = e\hbar/(2m) = 9.274 \times 10^{-24}$ J/T.

$N(E) = G(E)\,f(E)$. Boltzmann factor: $\exp[-E/(k_BT)]$. Gibbs factor: $\exp[(N\mu - E)/(k_BT)]$.

$f_{FD}(E) = \dfrac{1}{\exp[(E - E_F)/(k_BT)] + 1}$. Electron gas: $E_{F0} = \frac{h^2}{8m}\left(\dfrac{3N}{\pi V}\right)^{2/3}$, $C_V = \dfrac{9Nk_B^2}{2E_{F0}} T$

$f_{BE}(E) = \dfrac{1}{\exp[E/(k_BT)] - 1}$.

$\dfrac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3$. $B_{12} = B_{21}$. $k_B = 8.617 \times 10^{-5}$ eV/K.

$R = r_0 A^{1/3}$ with $r_0 = 1.2 \times 10^{-15}$ m.

$$\text{BE} = -a_1 A + a_2 A^{2/3} + a_3\frac{Z^2}{A^{1/3}} + a_4\frac{(N - Z)^2}{A}$$

with $a_1 = 15.5$ MeV, $a_2 = 16.8$ MeV, $a_3 = 0.72$ MeV, and $a_4 = 19$ MeV.

$N = N_0\exp[-\lambda t]$, activity $R = \lambda N$. 1 Gray=1 J/kg=100 rad. Sievert=Gray·RBE=100 rem. 1 Curie=3.7×10^{10} decay/sec. $N_A = 6.02 \times 10^{23}$ atoms/mol.

$\nu = \dfrac{\nu_0}{\gamma[1 + (v/c)\cos\theta]}$ with $\theta = 0$ for emitter moving directly away.
$\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$
$p = \gamma m_0 v$, $F_\perp = \gamma m_0 a$, $F_\parallel = \gamma^3 m_0 a$.

$$
\begin{aligned}
x_2 &= \gamma\left(x_1 - v t_1\right) \\
y_2 &= y_1 \\
z_2 &= z_1 \\
t_2 &= \gamma\left(t_1 - \frac{v}{c} x_1\right)
\end{aligned}
$$

and

$$
\begin{aligned}
v_{2,x} &= \frac{v_{1,x} - v}{1 - \dfrac{v\, v_{1,x}}{c^2}} \\
v_{2,y} &= \frac{v_{1,y}}{\gamma\left[1 - \dfrac{v\, v_{1,x}}{c^2}\right]} \\
v_{2,z} &= \frac{v_{1,z}}{\gamma\left[1 - \dfrac{v\, v_{1,x}}{c^2}\right]}
\end{aligned}
$$

$E = E_0 + E_k = m_0 c^2 + (\gamma - 1) m_0 c^2$, $E^2 = E_0^2 + p^2 c^2$.

$$
\begin{aligned}
p_{x,2} &= \gamma\left(p_{x,1} - v(E/c^2)\right) \\
p_{y,2} &= p_{y,1} \\
p_{z,2} &= p_{z,1} \\
E_2 &= \gamma(E - v p_x).
\end{aligned}
$$