

# Electrostatics and gravity

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford scattering

Experimental confirmation

Closest approach

- Recall electrostatic force:  $F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$  with  $\epsilon_0 = 8.85 \times 10^{-12}$  (mks units).
- Recall gravitational force:  $F = G \frac{m_1m_2}{r^2}$  with  $G = 6.67 \times 10^{-11}$  (mks units).
- Ratio is  $4\pi\epsilon_0 G \frac{m_1m_2}{q_1q_2}$ .
- Two protons:

$$4\pi\epsilon_0 G \frac{m_1m_2}{q_1q_2} = 4\pi\epsilon_0 G \frac{(1.7 \times 10^{-27})^2}{(1.6 \times 10^{-19})^2} = 8.1 \times 10^{-37}$$

- Two electrons:

$$4\pi\epsilon_0 G \frac{m_1m_2}{q_1q_2} = 4\pi\epsilon_0 G \frac{(9.1 \times 10^{-31})^2}{(1.6 \times 10^{-19})^2} = 2.4 \times 10^{-43}$$

- Gravity is a wimp! **Most things must be electrostatically neutral.**

# The discovery of the electron

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- The American Institute for Physics has a good [web site](#) on this.
- Remember that much was swirling about with cathode ray tubes: first the discovery of X rays, then the discovery of electrons, then the photoelectric effect, all within a decade.
- Cathode rays: drawn from regular matter (the cathode). Are they particles? Constituents of atoms? Do they have some elementary charge?
- J.J. Thomson, director of the Cavendish Lab at Cambridge, thought so, but wanted to prove this.

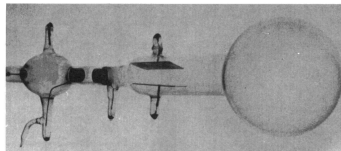


J.J. Thomson  
(1856–1940; Nobel  
Prize 1906)

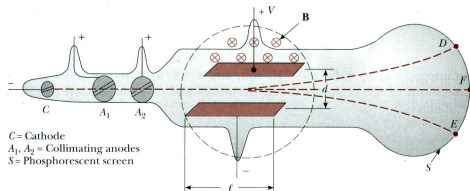
# Thomson's experiments

1. cathode rays carry negative charge (electrometer).
2. with good enough vacuum, electric field deflects cathode rays
3. use an accelerating voltage to give corpuscles with the same charge the same velocity

$$qV_x = (1/2)mv^2 \text{ or}$$
$$v = \sqrt{2qV_x/m}$$



**Figure 4.4** The original  $e/m_e$  tube used by J. J. Thomson. (After Figure 1.3, p. 7, R. L. Sproull and W. A. Phillips, *Modern Physics*, 3rd ed., New York, John Wiley & Sons, 1980).



C = Cathode  
A<sub>1</sub>, A<sub>2</sub> = Collimating anodes  
S = Phosphorescent screen

**Figure 4.5** A diagram of Thomson's  $e/m_e$  tube (patterned after J. J. Thomson, *Philosophical Magazine* (5)44:293, 1897). Electrons subjected to an electric field alone land at D, while those subjected to a magnetic field alone land at E. When both electric and magnetic fields are present and properly adjusted, the electrons experience no net deflection and land at F.

## Magnetic field alone

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Thomson's third experiment with a magnetic field alone:

$$m \frac{v^2}{r} = qvB \quad \Rightarrow \quad \frac{q}{m} = \frac{v}{rB}$$
$$\Rightarrow \quad \frac{q}{m} = \frac{\sqrt{2qV_x/m}}{rB} = \sqrt{\frac{q}{m}} \frac{\sqrt{2V_x}}{rB}$$

giving

$$\frac{q}{m} = \frac{2V_x}{r^2 B^2} \quad (1)$$

so I'm mystified why Thomson didn't consider this sufficient! Note that this ignores relativity; it would have us start out with  $\gamma m v^2 / r = qvB$  (1909).

## Electric field alone

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- Acceleration due to voltage  $V_y$  over plate separation  $d$ :

$$a_y = qE/m = qV_y/(md)$$

- Velocity in  $\hat{y}$  changes from 0 to

$$v_y = a_y \cdot \Delta t = \frac{qV_y}{md} \cdot \frac{\ell}{v_x} \quad (2)$$

- Recall velocity in  $\hat{x}$  direction:  $v_x = \sqrt{2qV_x/m}$
- Angle upon leaving electric field region:

$$\begin{aligned} \theta &\simeq \tan \theta = \frac{v_y}{v_x} = \frac{qV_y}{md} \frac{\ell}{v_x} \frac{1}{v_x} \\ &= \frac{q}{m} \frac{\ell V_y}{dv_x^2} \\ &= \frac{qV_y}{md} \frac{\ell m}{2qV_x} = \frac{\ell V_y}{2dV_x} \end{aligned}$$

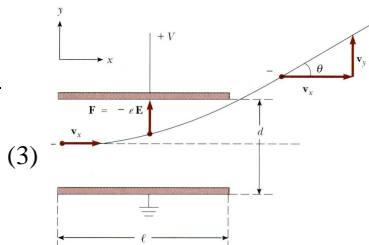


Figure 4.6 Deflection of negative particles by an electric field.

which tells us nothing about  $q$  or  $m$ .

## Electric and magnetic fields both

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- If we start from the electric field result of Eq. 3 of

$$\theta = \frac{q}{m} \frac{\ell V_y}{dv_x^2}$$

and add in an orthogonal magnetic field to cancel out the electric field direction, we then have

$$qv_x B = qE \quad \text{so} \quad v_x = \frac{E}{B} = \frac{V_y}{Bd} \quad (4)$$

- We can use this result of Eq. 4 in the expression of Eq. 3:

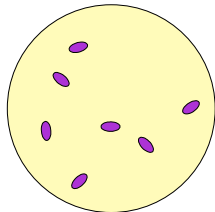
$$\theta = \frac{q}{m} \frac{\ell V_y}{dv_x^2} = \frac{q}{m} \frac{\ell V_y}{dV_y^2} B^2 d^2 = \frac{q}{m} \frac{\ell dB^2}{V_y}$$

giving 
$$\frac{q}{m} = \frac{\theta V_y}{\ell dB^2} \quad (5)$$

which also gives us a way to measure the ratio  $q/m$  (Serway Eq. 4.7). Why is this better than the magnetic-field-alone result of our Eq. 1? I'm not sure. . . Maybe Thomson felt it was easier to measure  $\theta$  rather than  $r$ ?

# Thomson's plum pudding model

- Atoms, like nearly all matter, are electrically neutral.
- “Corpuscles” of negative charge have known  $e/m$  ratio. Since remaining bit of hydrogen atom is equally positively charged, and mass of atom could be estimated, Thomson inferred that electrons have equal but opposite charge and thus carry only  $1/800^{\text{th}}$  of the mass.
- Therefore bulk of atom must be positive with little negative bits inside: the plum pudding model.
- Post-Einstein: “classical” radius of the electron  $r_e = 2.82 \times 10^{-15}$  meters. This equates energy required to shrink an electron charge from a radius of infinity down to a radius at which electrostatic energy stored is equal to  $mc^2$  rest energy. Compare with Einstein's Brownian motion estimate of atom sizes of about  $2 \times 10^{-10}$  meters.



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# Elementary charge

Forces:  $q$  and  $m$

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- R.A. Millikan and H. Fletcher at the University of Chicago, 1909: Charge always comes in integer multiples of a basic value. So now we really have  $e$  pinned down, and therefore we also know  $m_e$ !
- Millikan also carried out further studies of the photoelectric effect, confirming Einstein's prediction and obtaining an improved value for Planck's constant  $h$  during the time period 1912–1915.



R.A. Millikan (1868–1953; Nobel Prize 1923)

## Indigestion from plum pudding

Displacing an electron by  $r$  from the center of an atom of radius  $R$  and positive charge  $Z$  gives a restoring force of

$$F = \frac{Ze^2}{4\pi\epsilon_0 R^3} r \quad (6)$$

(see Krane Eq. 6.1) which has the form  $F = kr$  with  $k \equiv Ze^2/(4\pi\epsilon_0 R^3)$ . Therefore for hydrogen ( $Z = 1$ ) we have a harmonic oscillator with a resonant frequency of

$$\begin{aligned} \nu &= \frac{1}{2\pi} \sqrt{k/m} = \frac{1}{2\pi} \frac{e}{\sqrt{4\pi\epsilon_0 R^3 m_e}} \\ &= \frac{1}{2\pi} \frac{1.602 \times 10^{-19}}{\sqrt{4\pi \cdot 8.854 \times 10^{-12} \cdot (1 \times 10^{-10})^3 \cdot 9.109 \times 10^{-31}}} \\ &= 2.5 \times 10^{15} \text{ Hz} \end{aligned}$$

This corresponds to a radiation wavelength of  $\lambda = c/\nu = 120 \text{ nm}$ .

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## Meanwhile... Radioactivity!

Forces:  $q$  and  $m$

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- Early 1896: Henri Becquerel noticed that uranium compounds would fog photographic plates—the discovery of radioactivity.
- 1898: Marie Sklodowska Curie measures radioactivity by looking at ionization of air. Unaffected by chemical binding, heat, etc.! Husband Pierre then joins research; they discover radium and polonium.
- Radioactive decay releases energies in the MeV range!!!
- Becquerel, and Marie and Pierre Curie share the 1903 Nobel Prize in Physics. Marie is awarded the 1911 Nobel Prize in Chemistry.



Antoine Henri  
Becquerel  
(1852–1908)



Marie Curie  
(1867–1934) and

Pierre Curie

## Enter Rutherford: the alpha male

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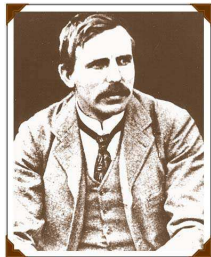
Thomson scattering

Rutherford scattering

Experimental confirmation

Closest approach

- Grew up on a farm in New Zealand, and studied at the University there. Applied for a graduate scholarship at Cambridge and worked at home while awaiting a reply. When the scholarship letter came (1894), he threw down his shovel and said “That’s the last potato I will ever dig.”
- McGill University in Montreal, 1898–1907. University of Manchester, 1907–1919. Cavendish Professor at Cambridge, 1919–1937.
- “All science is either physics or stamp collecting”



Ernest Rutherford  
(1871–1937; Nobel Prize in Chemistry, 1908)

# Discovery of the alpha particle

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Rutherford at McGill in Montreal, 1905:

- Fill thin-glass-wall tube with radon, which emits alpha particles ( $\alpha$ ).
- Surround that tube with another thick-walled, evacuated tube.
- After a few days, helium identified in the outer tube by its characteristic spectrum.
- Alpha particles are helium nuclei (2 protons, 2 neutrons, or  ${}^4_2\text{He}$ , using  ${}^A_n\text{X}_Z$ ).
- Other radioactive decay emissions:  $\beta^-$  are electrons,  $\gamma$  are very energetic photons.

1908 Nobel Prize in Chemistry.

# Rutherford's $\alpha$ apparatus

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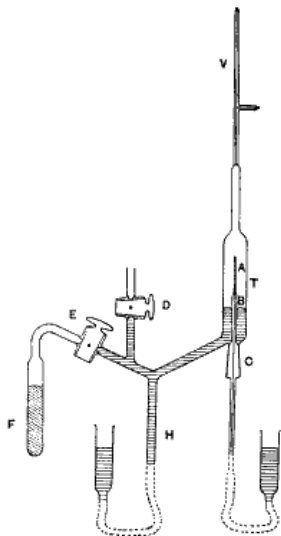
**Rutherford**

Thomson scattering

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Experimental confirmation

Closest approach



# Radioactivity: lots of energy!

- Rutherford and Soddy, 1903: “The energy of radioactive change must therefore be at least twenty-thousand times, and may be a million times, as great as the energy of any molecular change.”  
*Remember that chemistry happens at  $\sim 3\text{--}10\text{ eV}$ , while nuclear decays happen at  $10^6\text{ eV}$ .*

Quoted in Rhodes, *The making of the atomic bomb*

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- Soddy in 1904: “If it [the energy of the nucleus] could be tapped and controlled what an agent it would be in shaping the world’s destiny! The man who put his hand on the lever by which a parsimonious nature regulates so jealously the output of this store of energy would possess a weapon by which he could destroy the earth if he chose.”

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- Soddy continued: “The fact that we exist is a proof that [massive energetic release] did not occur; that it has not occurred is the best possible assurance that it never will. We may trust Nature to guard her secret.”

Quoted in Rhodes, *The making of the atomic bomb*

## Other gems from [www.wikiquote.org](http://www.wikiquote.org)

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- “The energy produced by the breaking down of the atom is a very poor kind of thing. Anyone who expects a source of power from the transformation of these atoms is talking moonshine.”—Ernest Rutherford, shortly after splitting the atom for the first time.

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- “Nuclear-powered vacuum cleaners will probably be a reality in 10 years.”—Alex Lewyt, president of vacuum cleaner company Lewyt Corp., in the New York Times in 1955.

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plus

- “It’s hard to make predictions—especially about the future.”—attributed to Robert Storm Petersen.

# Use $\alpha$ particles to probe the nucleus

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Experimental confirmation

Closest approach

- Radium emits  $\alpha$  particles with 5 MeV kinetic energy.
- Alpha mass is (slightly less than)  
 $m = 2 \cdot 938.3 + 2 \cdot 939.6 = 3757.8 \text{ MeV}/c^2$  (2 protons, 2 neutrons)
- Therefore  $E_k = (\gamma - 1)mc^2$  or  
 $\gamma = 1 + E_k/mc^2 = 1 + 5/3.75 = 2.33$  and  
 $\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/2.33^2} = 0.90$ .
- Gold nucleus significantly outweighs  $\alpha$ . Gold can be hammered into very thin foils. Interatomic spacing:

$$\begin{aligned} \left( \frac{A}{\rho N_A} \right)^{1/3} &= \left( \frac{197 \text{ g/mol}}{(18.9 \text{ g/cm}^3) \cdot (6.02 \times 10^{23} \text{ mol}^{-1})} \right)^{1/3} \\ &= 2.6 \times 10^{-8} \text{ cm} \end{aligned}$$

## $\alpha$ backscattering?

Forces:  $q$  and  $m$

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**Rutherford**

Thomson scattering

Rutherford  
scattering

Experimental  
confirmation

Closest approach

- So  $\alpha$  particles have  $\beta \simeq 0.90$  and mass of about  $4/197 \simeq 0.020$  that of gold nuclei.
- While at McGill, Rutherford had noticed significant scattering at large angles. That seemed odd. . .
- After receiving Nobel Prize, Rutherford is offered position at Manchester in U.K. Wants to look into  $\alpha$  scattering in more detail.
- What would one expect for Thomson's plum pudding model?

# Thomson scattering I

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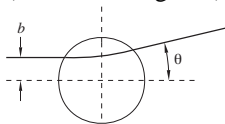
- Tiny bullet on a plum pudding atom is explained by Thomson scattering.
- Particle of charge  $ze$  approaches a sphere of charge  $Ze$  in a radius  $R$ , with an impact parameter (distance off from the centerline) of  $b$ .
- For small  $\theta$ , distance particle travels through sphere is chord  $c$  at  $b$ , or

$$b^2 + \left(\frac{c}{2}\right)^2 = R^2 \text{ or } c = 2\sqrt{R^2 - b^2}.$$

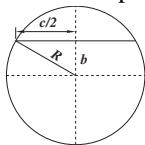
- Time that particle spends inside the sphere is (Krane Eq. 6.5)

$$T = \frac{c}{v} = 2\frac{\sqrt{R^2 - b^2}}{v} \quad (7)$$

Scattering geometry (like Krane Fig. 6.3):



Chord of sphere:



## Thomson scattering II

- The force on a charge  $q_1$  displaced by a distance  $r$  from the center of a uniformly distributed charge of  $q_2$  in a radius  $R$  is given by

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 R^3} r \quad (8)$$

with  $q_1 = ze$  for the  $\alpha$  particle, and  $q_2 = Ze$  for the gold nucleus.

- Therefore the vertical force due to offset  $b$  is nearly constant over the chord  $c$ :

$$F_y \simeq kzb \quad (9)$$

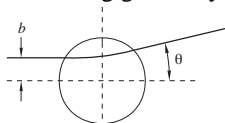
with

$$k \equiv \frac{Ze^2}{4\pi\epsilon_0 R^3} \quad (10)$$

- Momentum impulse transferred to the scattered particle depends on time  $T = c/v$  from Eq. 7:

$$\Delta p_y = \int F_y dt = \int kzb dt = kzbT = \frac{2kzb}{v} \sqrt{R^2 - b^2}. \quad (11)$$

Scattering geometry:



# Thomson scattering III

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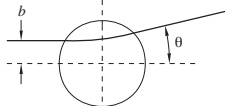
Experimental confirmation

Closest approach

- Again, momentum impulse  $\Delta p_y$  is (Eq. 11)

$$\Delta p_y = \frac{2kzb}{v} \sqrt{R^2 - b^2}$$

Scattering geometry:



- Scattering angle  $\theta$  in classical limit ( $p = mv$ ,  $E_k = mv^2/2$ ) is then

$$\theta \simeq \frac{\Delta p_y}{p} = \frac{2kzb}{mv^2} \sqrt{R^2 - b^2}. \quad (12)$$

- For  $b = R/2$  and using Eqs. 8 and 10, we have (like Krane Eq. 6.9)

$$\theta_{\text{typ.}} = 2k \frac{z(R/2)}{2E_k} R \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{4} \frac{Ze^2}{4\pi\epsilon_0 R^3} \frac{zR^2}{2E_k} = \frac{\sqrt{3} zZe^2}{16\pi\epsilon_0 R E_k} \quad (13)$$

# Thomson scattering IV

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Rutherford

**Thomson scattering**

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scattering

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confirmation

Closest approach

- Use  $z = 2$ ,  $Z = 79$ ,  $R = 2.6 \times 10^{-10}$  m,  
 $E_k = (5 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV}) = 8.0 \times 10^{-13}$  J to obtain

$$\begin{aligned}\theta_{\text{typ.}} &= \frac{\sqrt{3} z Z e^2}{16\pi\epsilon_0 R E_k} \\ &= \frac{\sqrt{3} \cdot 2 \cdot 79 \cdot (1.602 \times 10^{-19})^2}{16\pi \cdot (8.854 \times 10^{-12}) \cdot (2.6 \times 10^{-10}) \cdot (8.0 \times 10^{-13})} \\ &= 7.6 \times 10^{-5} \text{ rad} = 0.076 \text{ mrad} \quad (14)\end{aligned}$$

# Rutherford's suggestion

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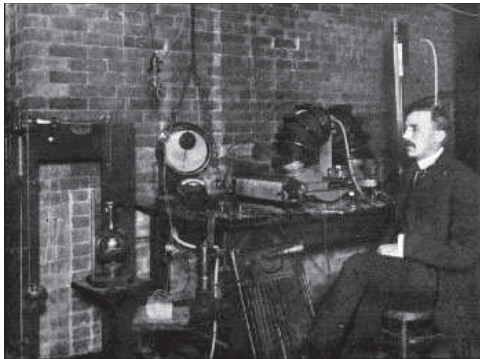
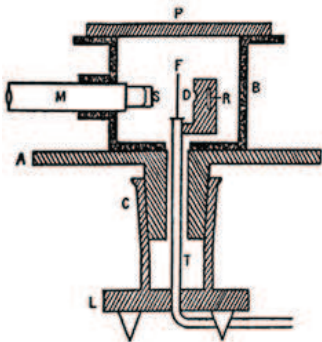
Experimental  
confirmation

Closest approach

- So. . . What's inside the bowl of plum pudding? Let's shoot bullets of  $\sim 8000 \times$  mass of electron to find out!
- Gold hammered to  $2 \mu\text{m}$  thickness: about  $10^4$  atoms thick.
- Net deflection angle for  $N$  uncorrelated scatterings of value  $\theta$  each is  $\sqrt{N}\theta$
- Thus for gold foil we expect about  $100\theta_{\text{typ.}}$ , or about 8 mrad.
- Not at all consistent with Rutherford's earlier rough observations! Look into it more closely.
- Rutherford assigns experiment to Ernest Marsden (age 20) and Hans Geiger. They use a microscope focused on a ZnS screen to observe flashes of light from single  $\alpha$  particles.

## The experiment

Set the Radium  $\alpha$  emitter R and foil F to one angle. Sit in the dark for several minutes to become dark-adapted (Rutherford would tell stories). Stare into the microscope M and count flashes of light on the zinc sulfide screen S for a long time. . .



# The unexpected

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford  
scattering

Experimental  
confirmation

Closest approach

- Marsden and Geiger find significant scattering at large angles, including back towards the  $\alpha$  particle source! Rutherford:  
“It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration I realised that this scattering backwards must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greatest part of the mass of the atom was concentrated in a minute nucleus.”
- Context: *HMS Dreadnought* (1906) with 12 inch guns. *HMS Queen Elizabeth* (1913) with 15-inch guns: 1900 lb. shells, muzzle velocity of 1700 mph, range of 11 miles!

# Rutherford's interpretation

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford scattering

Experimental confirmation

Closest approach

- Return to Eq. 13 of

$$\theta_{\text{typ.}} = \frac{\sqrt{3} zZe^2}{8\pi\epsilon_0 R E_k}$$

- We need to scale up from single scattering of 0.08 mrad to 1.5 rad, or a factor of  $\sim 10^4$  (and another factor of about 2).
- What's the chance of having *all*  $10^4$  scattering events give  $\theta > \theta_{\text{typ.}}$ ? Assume probability of (1/2) for each scatter to be larger than  $\theta_{\text{typ.}}$ ; net probability is then  $(1/2)^{10^4}$ . If we have  $A = B^C$ , then

$$\log_{10} A = \log_{10} B^C = C \log_{10} B,$$

$$\text{so } (1/2)^{10^4} = 10^{10^4 \log_{10}(1/2)} = 10^{-3000}.$$

- To do this by  $N$  uncorrelated scatters, we would need  $\sqrt{N} = 10^4$  or  $N = 10^8$  or a foil thickness of  $10^8 \cdot 2.6 \times 10^{-10} \text{ m} = 2.6 \text{ cm}$ . Ludicrous!
- What can we change?  $R$  is the only thing! Must scale it down by  $10^{-4}$ , from  $2.6 \times 10^{-10}$  to  $2.6 \times 10^{-14}$ .

## Re-examine the situation

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford scattering

Experimental confirmation

Closest approach

- Conclusion: the nucleus is *very* small. When Rutherford figures it out (early 1911), he marches into Geiger's office humming "Onward Christian Soldiers" and announces, "I know what the atom looks like!"
- With a point-like nucleus, electrons must be in some sort of orbital motion.
- Now we re-think the problem, with  $R \ll b$ . It can be shown that a particle follows a hyperbolic path when it passes near a point source of a  $1/r^2$  repulsive force. For Coulomb repulsion, one obtains (Krane Eq. 6.12)

$$\frac{1}{r} = \frac{1}{b} \sin \varphi + \frac{zZe^2}{8\pi\epsilon_0 b^2 E_k} (\cos \varphi - 1), \quad (15)$$

where  $E_k$  is the kinetic energy of the alpha particle.

- For scattering angles of  $\theta = \pi - \varphi$ , one can solve for  $b$  to obtain (Krane Eq. 6.13)

$$b = \frac{zZe^2}{8\pi\epsilon_0 E_k} \cot\left(\frac{1}{2}\theta\right). \quad (16)$$

## Rutherford scattering II

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford scattering

Experimental confirmation

Closest approach

- Fraction  $f$  of scatterings at an angle of  $\theta$  or larger will be given by the fractional area taken up by atoms with impact parameter  $b$  in a foil of thickness  $t$ ; this can be found to be (Krane Eq. 6.15)

$$f_{>\theta} = \frac{\rho N_A}{A} t \pi b^2. \quad (17)$$

- To find the range of impact parameters  $b$  to  $b + db$  which produce scattering within an angular range  $d\theta$ , we must find

$$d(f_{>\theta}) = \frac{N_A \rho}{A} t (2\pi b db). \quad (18)$$

- The  $db$  term can be found from Eq. 16 to be (Krane Eq. 6.16)

$$db = \frac{zZe^2}{8\pi\epsilon_0 E_k} \left(-\csc^2\left[\frac{1}{2}\theta\right]\right) \left(\frac{1}{2}d\theta\right), \quad (19)$$

which gives (Krane Eq. 6.17)

$$d(f_{>\theta}) = \frac{N_A \rho}{A} \pi t \left(\frac{zZe^2}{8\pi\epsilon_0 E_k}\right)^2 \csc^2\left(\frac{1}{2}\theta\right) \cot\left(\frac{1}{2}\theta\right) d\theta. \quad (20)$$

# Rutherford scattering III

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford  
scattering

Experimental  
confirmation

Closest approach

- In fact, our detector is not likely to collect *all* events at an angle larger than  $\theta$ ; instead, we will consider the fraction of events which fall in a ring at a distance  $r$  from the scatterer which collects an angular range of  $d\theta$  about  $\theta$ .
- The detector area integrated over  $2\pi$  azimuthally, the radius of the  $d\theta$  ring is  $r \sin \theta$ , and the width of the ring is  $r d\theta$ , so the detector area is  $2\pi r^2 \sin \theta d\theta$ .
- The fraction  $N(\theta)$  of events we expect to detect can then be shown to be (Krane Eq. 6.18)

$$N(\theta) = \frac{N_A \rho}{A} \frac{t}{4r^2} \left( \frac{zZe^2}{8\pi\epsilon_0 E_k} \right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}. \quad (21)$$

## Angular distribution

Agreement between Rutherford's theory and Marsden's experiments  
(Krane Fig. 6.14):

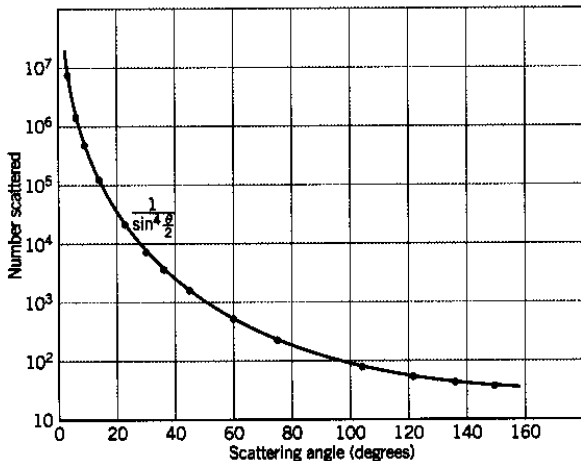


FIGURE 6.14 The dependence of scattering rate on the scattering angle  $\theta$ , using a gold foil. The  $\sin^{-4}(\theta/2)$  dependence is exactly as predicted by the Rutherford formula.

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford

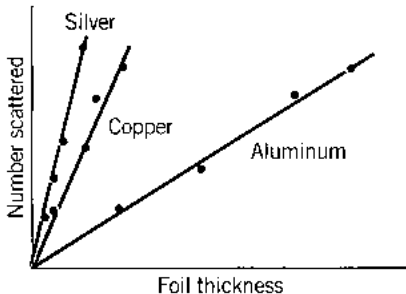
scattering

Experimental  
confirmation

Closest approach

## Effect of foil thickness

Rutherford predicts that  $N(\theta) \propto t$ , the foil thickness, whereas recall that the Thomson model would have predicted  $N(\theta) \propto \sqrt{t}$ . See Krane Fig. 6.11:



**FIGURE 6.11** The dependence of scattering rate on foil thickness for three different scattering foils.

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

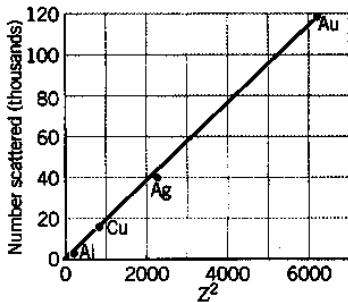
Rutherford scattering

Experimental confirmation

Closest approach

## Effect of atomic number $Z$

Rutherford predicts  $N(\theta) \propto Z^2$ , which stands in contrast to the linear dependence on  $Z$  in Eq. 13. See Krane Fig. 6.12:



**FIGURE 6.12** The dependence of scattering rate on the nuclear charge  $Z$  for foils of different materials. The data are plotted against  $Z^2$ .

## Distance of closest approach

Forces:  $q$  and  $m$

The electron

Plum pudding

Electron charge

Radioactivity

Rutherford

Thomson scattering

Rutherford  
scattering

Experimental  
confirmation

Closest approach

- In backscattering, the kinetic energy of the alpha particle must be converted completely into electrostatic potential energy at the point of closest approach:

$$E_k = \frac{1}{4\pi\epsilon_0} \frac{zZe^2}{r_{\min}} \quad \Rightarrow \quad r_{\min} = \frac{1}{4\pi\epsilon_0} \frac{zZe^2}{E_k}, \quad (22)$$

which for a 5 MeV alpha on gold gives  $r_{\min} = 5 \times 10^{-14}$  m.

- In fact, an absolute measurement of  $N(\theta)$  was shown to be consistent with a radius of the nucleus which is about  $1/10^{\text{th}}$  of this value, which is why the alpha particle does not induce fission in gold.