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Review: momentum and forces

- We found that relativistic momentum goes like $p_1 = \gamma m_0 v_2$
- For perpendicular forces ($\vec{F} \perp \vec{v}$) we found $\vec{F} = \gamma m_0 \vec{a}$
  - The centripetal force for uniform circular motion is $F = \gamma m v^2 / r$.
- For parallel forces ($\vec{F} \parallel \vec{v}$) we found $\vec{F} = \gamma^3 m_0 \vec{a}$

Review: relativistic energy

- Relativistic kinetic energy is $E_k = (\gamma - 1)m_0c^2$.
  - Classical limit reduces to $E_k = \frac{1}{2}mv^2$
- Suggests total energy of

$$E_{\mathrm{tot}} = E_0 + E_k = m_0c^2 + (\gamma - 1)m_0c^2 = \gamma m_0c^2 \tag{1}$$

- Common unit for energies in modern physics: electron·Volt, where 1 eV=1.602×10^{-19} Joules. Chemical bonds are typically 3–6 eV.
- Can describe particle masses in energy units.
  - Electron: $m_e = 511 \times 10^3$ eV/c^2 or just 511 keV.
  - Proton: $m_p = 939 \times 10^6$ eV/c^2 or just 939 MeV.

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Relativistic conservation of momentum

- Classically, kinetic energy is $p^2/2m$. Consider p^2 in relativity:

$$(pc)^2 = (\gamma m_0 v c)^2 = (\gamma \beta m_0 c^2)^2. \tag{2}$$

- If we then use $E_0 = m_0 c^2$ and $\beta^2 = 1 - \frac{1}{\gamma^2}$, we obtain

$$p^2c^2 = \gamma^2(1 - \frac{1}{\gamma^2})E_0^2 = (\gamma^2 - 1)E_0^2 = \gamma^2 E_0^2 - E_0^2. \tag{3}$$

  However, since we found before that the total energy is $E = \gamma E_0$, we have $p^2c^2 = E^2 - E_0^2$ or

$$E^2 = E_0^2 + p^2c^2. \tag{4}$$

- Therefore if $E_k \gg E_0$ we have $E_{k,\text{relativistic}} \simeq pc$.

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Mass of the photon

- What is the mass of something that is allowed to travel at the speed of light?
- Eq. 1 was $E = E_0 + E_k = \gamma m_0 c^2 = \gamma E_0$
- If we rewrite this as

$$E_0 = \frac{E}{\gamma} = E\sqrt{1 - v^2/c^2}. \qquad (5)$$

  we see that if we set $v = c$, we get $E_0 = E\sqrt{1-1} = 0$.
- If $E_0 = 0$ then $m_0 = 0$!
- Because light must travel with a velocity of c, we therefore conclude that photons have a rest mass of zero.
- You can't halt the photon! A stationary photon isn't.

Energy–momentum invariant

Rearrange Eq. 4 of $p^2c^2 = \gamma^2 E_0^2 - E_0^2$ to give

$$\left(\frac{E_0}{c}\right)^2 = \left(\frac{E}{c}\right)^2 - p^2 = \left(\frac{E}{c}\right)^2 - p_x^2 - p_y^2 - p_z^2. \quad (6)$$

Now $(E_0/c)^2 = (m_0c)^2$ has the same value when measured in any inertial frame; therefore so does the quantity on the left hand side of Eq. 6. Thus

$$\left(\frac{E_1}{c}\right)^2 - p_{x,1}^2 - p_{y,1}^2 - p_{z,1}^2 = \left(\frac{E_2}{c}\right)^2 - p_{x,2}^2 - p_{y,2}^2 - p_{z,2}^2 \quad (7)$$

between frames S_1 and S_2. This is really the equivalent of our statement in a previous lecture

$$(ct_1)^2 - x_1^2 - y_1^2 - z_1^2 = (ct_2)^2 - x_2^2 - y_2^2 - z_2^2 \quad (8)$$

which served as the basis for our derivation of the Lorentz transformations!

Momentum transforms

We went from $(ct_1)^2 - x_1^2 - y_1^2 - z_1^2 = (ct_2)^2 - x_2^2 - y_2^2 - z_2^2$ to find transformations for position and time:

$$\begin{aligned} x_2 &= \gamma\,(x_1 - vt_1) \qquad \text{and} \qquad y_2 = y_1 \qquad \text{and} \qquad z_2 = z_1 \\ t_2 &= \gamma\left(t_1 - \frac{\beta}{c}x_1\right) \end{aligned}$$

From Eq. 7 of $(E_1/c)^2 - p_{x,1}^2 - p_{y,1}^2 - p_{z,1}^2 = (E_2/c)^2 - p_{x,2}^2 - p_{y,2}^2 - p_{z,2}^2$ we can find an equivalent Lorentz transform for momentum and energy:

$$\begin{aligned} p_{x,2} &= \gamma\left(p_{x,1} - v\left(\frac{E_1}{c^2}\right)\right) \text{ and } p_{y,2} = p_{y,1} \text{ and } p_{z,2} = p_{z,1} \qquad (9) \\ E_2 &= \gamma(E_1 - vp_{x,1}). \end{aligned}$$

This is somewhat startling, for it tells us that we need to worry about the Lorentz transformation in considering conservation of energy!

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Relativisic collision I

A particle A with rest mass m_0 and velocity $v_A = 0.80c$ in the $\hat{x}$ direction collides with an initially-stationary particle B with rest mass $2m_0$. Note that for $\beta = 4/5$ we can find $\gamma = \frac{5}{3}$. In the frame S_1, we then use $p_{1,y} = \gamma m_0 v_{2,y}$ and $E = \gamma m_0 c^2$ to find

$$\begin{aligned} p_{x,A,1} &= \gamma m_0 v_{x,A} = \frac{5}{3} m_0 \frac{4}{5} c = \frac{4}{3} m_0 c. \\ p_{y,A,1} &= p_{z,A,1} = 0 \\ E_{A,1} &= \gamma m_0 c^2 = \frac{5}{3} m_0 c^2 \end{aligned}$$

for particle A, and

$$\begin{aligned} p_{x,B,1} &= \gamma m_0 v_{x,B} = 0 \\ p_{y,B,1} &= p_{z,B,1} = 0 \\ E_{B,1} &= \gamma m_0 c^2 = 1(2m_0)c^2 \end{aligned}$$

for particle B.

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Relativistic collision II

The total energy in frame S_1 is then

$$E_1 = E_{A,1} + E_{B,1} = \left(\frac{5}{3} + 2\right) m_0 c^2 = \frac{11}{3} m_0 c^2,$$

or $E_1 = 3.67 m_0 c^2$.
Chose center-of-momentum frame. Using $p_{x,2} = \gamma\left(p_{x,1} - v(E/c^2)\right)$, we find

$$\begin{aligned} p_{x,A,2} + p_{x,B,2} &= 0 = \gamma\left[\left(p_{x,A,1} - v(E_{A,1}/c^2)\right) + \left(p_{x,B,1} - v(E_{A,1}/c^2)\right)\right] \\ &= \frac{\left(\frac{4}{3}m_0 c - \frac{5}{3}m_0 v\right) + \left(0 - 2m_0 v\right)}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

The result of the final line will be zero if the numerator on the final line is zero, or $\left(\frac{4}{3}c - \frac{5}{3}v - 2v\right) m_0 = 0$ from which we obtain a velocity v of S_2 relative to S_1 of $v = (4/11)c$.

Relativistic collision III

The total energies of the individual particles in S_2 can be found using $E_2 = \gamma(E_1 v p_{x,1})$ to be

$$
\begin{aligned}
E_{A,2} &= \frac{E_{A,1} - v p_{x,A,1}}{\sqrt{1 - v^2/c^2}} \\
&= \frac{\frac{5}{3} m_0 c^2 - \left(\frac{4}{11} c\right)\left(\frac{4}{3} m_0 c\right)}{\sqrt{1 - \left(\frac{4}{11}\right)^2}} = 1.27 m_0 c^2 \\
E_{B,2} &= \frac{E_{B,1} - v p_{x,B,1}}{\sqrt{1 - v^2/c^2}} \\
&= \frac{2 m_0 c^2 - 0}{\sqrt{1 - \left(\frac{4}{11}\right)^2}} = 2.15 m_0 c^2,
\end{aligned}
$$

or $E_2 = 3.24 m_0 c^2$. Thus total energy available in the center-of-mass frame is less than the total energy in the fixed-target frame.
To increase total energy in a collision, use center-of-mass frame when relativistic effects are considered.

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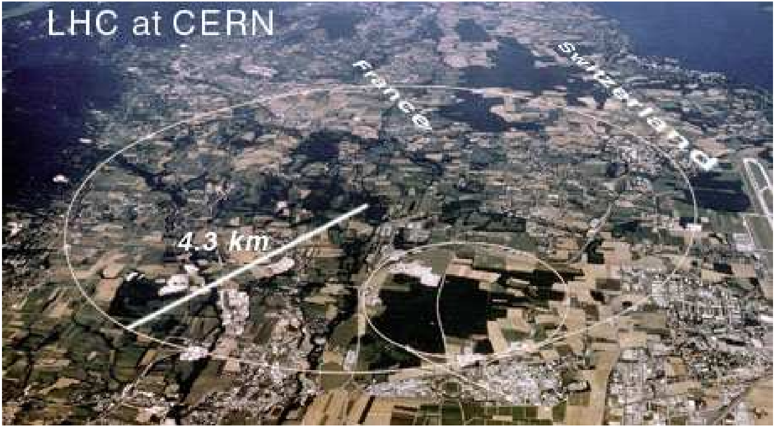
Large Hadronic Collider or LHC

The next (last?) large accelerator for subatomic particle physics is nearing completion at CERN (acronym originally stood for Conseil Européen pour la Recherche Nucléaire) in Geneva. 7 TeV (7×10^{12} eV) protons against 7 TeV anti-protons.



LHC on YouTube

- The LHC rap (an example of graduate students with too much time on their hands while waiting for the thing to get built): http://www.youtube.com/watch?v=j50ZssEojtM
- And here's a video of LHC making a black hole: http://www.youtube.com/watch?v=kVsZdgz5oFM

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ATLAS at the LHC

Several Stony Brook faculty are involved in experiments that will use this detector for capturing proton—anti-proton collisions at LHC. Notice the size of the people in this computer rendering?