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Comments on lecture notes

- On Tuesday, I rearranged slides “on the fly” during the lecture. As a result, the file `l1.pdf` as it now exists on the course web page is different than the file `l1.pdf` as it existed before that lecture. I might do this sort of thing quite frequently.
- Good ways to use the on-line lecture notes:
  - Save yourself the bother of writing down all the equations.
  - Concentrate on discussion rather than transcription.
- Bad ways to use the on-line lecture notes:
  - Stop coming to class. You’ll miss all the discussion and amplification of the notes!

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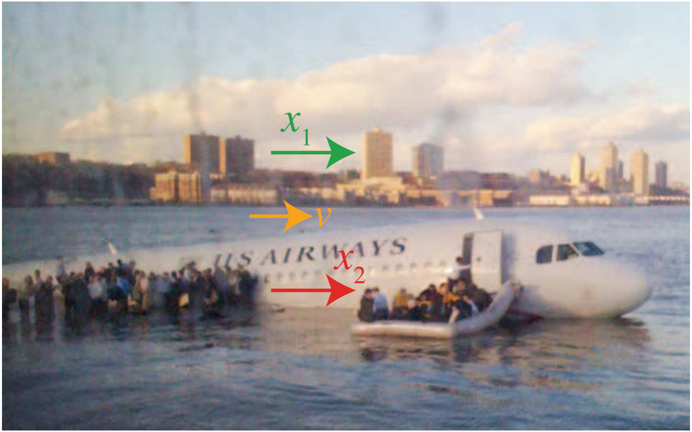
Feynman's Messenger Lectures

Richard Feynman was a co-recipient of the 1965 Nobel Prize in Physics for his role in the development of Quantum Electrodynamics, and gained fame as a physicst, a teacher, and as a colorful character. Thanks to Anthony Tricarichi for pointing out that some of his lectures are now available online:
http://research.microsoft.com/apps/tools/tuva/

Classical relativity

Let's pretend that we're in an Airbus A320 that has just landed in the Hudson. Observers on the shore are in frame S_1, while we on the airplane are in frame S_2 that's moving at a velocity $+v$ in the $\hat{x}$ direction relative to those on the shore.



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Classical relativity II

- Again, observers on the shore are in frame S_1, while we on the airplane are in frame S_2 that's moving at a velocity $+v$ in the $\hat{x}$ direction relative to those on the shore.
- Compare shore frame positions (x_1, y_1, z_1) with airplane frame positions (x_2, y_2, z_2).

  > "The frame S_2 we constrain to lie mainly on the plane."
  > (Apologies to Professor Henry Higgins and Eliza Doolittle in *My Fair Lady*)

- As the airplane floats downriver, we have a simple relationship between $\hat{x}$ positions in their frame S_1 versus our frame S_2:

$$\begin{aligned} x_2 = x_1 - vt \quad &\text{and} \quad y_2 = y_1 \text{ and} \\ z_2 = z_1 \quad &\text{and} \quad t_2 = t_1 \end{aligned} \tag{1}$$

  That is, when we stand on the wing of the airplane, it looks to us like people on shore are moving towards the tail.

- We also have a simple relationship between velocities in the two frames:

$$v_{2x} = v_{1x} - v \text{ and } v_{2y} = v_{1y} \text{ and } v_{2z} = v_{1z} \tag{2}$$

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Measuring a speedy meter stick

- Galilean relativity: simple addition of velocities.
- Let's use Galilean relativity to measure the length of a meter stick. We can certainly snap a photo of it as it goes flying by, and measure it from the photo!
- Let's say it has a velocity of $-v$ relative to us. Mark the positions of the two ends A and B:

$$\begin{aligned} A_2 &= A_1 + vt \\ B_2 &= B_1 + vt \\ B_2 - A_2 &= B_1 + vt - (A_1 + vt) = B_1 - A_1 \end{aligned}$$

  The length depends neither on v nor on t.
- According to Galilean relativity, we should see the length of the meter stick as being the length of the meter stick, no matter how quickly it's flying past us. But if A is under our nose and B is an arm's length away, don't we have to worry about the time delay for light to reach our eye from end B?

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Classical relativity: always?

- Saying that $x_2 = x_1 - vt$ and $v_{2x} = v_{1x} - v$ makes perfect sense in classical physics.
- Eminently reasonable when v describes the velocities of rivers (~ 1 m/s), cars (~ 30 m/s), jet airplanes and bullets (~ 300 m/s), and so on.
- But what about at velocities approaching $c = 3 \times 10^8$ m/s? If a spaceship traveling at half the speed of light aims a laser beam ahead, should we see light traveling at a velocity of $1.5c$?

Maxwell's silver hammer

Electromagnetic wave velocity: $c = 1/\sqrt{\mu_0 \epsilon_0}$. No equivocation!

> *This velocity is so nearly that of light, that it seems we have strong reasons to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.*

James Clerk Maxwell (1831–1879)

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Einstein's 3rd paper: motivation

First page of Einstein's 1905 paper "On the Electrodynamics of Moving Bodies" (translation from Griffiths, *Introduction to Electrodynamics*, Prentice-Hall, 1981, p. 392):

> *It is known that Maxwell's electrodynamics–as usually understood at the present time–when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. . .*
>
> *Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relative to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest.*

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Einstein's postulates

Special relativity: two frames with velocity difference (general relativity deals with large acceleration differences). Postulates:

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light in free space has the same value $c = 1/\sqrt{\mu_0 \epsilon_0}$ in all inertial reference frames.

Postulate 2 explains Michelson-Morley. But what about the speed of light coming out from the headlights of a spaceship?

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Light pulse from a point

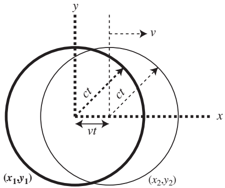
At $t_1 = t_2 = 0$, a spherical wave light pulse is emitted from a common source point. Frame S_2 is moving at a speed v in the $\hat{x}$ direction relative to frame S_1. Positions of pulse at a later time:

$$x_1^2 + y_1^2 + z_1^2 - c^2 t_1^2 = 0 \quad (3)$$
$$x_2^2 + y_2^2 + z_2^2 - c^2 t_2^2 = 0. \quad (4)$$

Applying Galilean relativity of

$$x_2 = x_1 - vt_1$$
$$y_1 = y_2$$
$$z_1 = z_2$$
$$t_1 = t_2 \quad (5)$$

gives inconsistencies! (Homework)



Observers in both frames see light pulse as a sphere expanding from an origin at the same velocity

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How to fix it?

Motion is in $\hat{x}$, so assume $y_1 = y_2$ and $z_1 = z_2$ as before. Simplest fix is a linear correction factor γ:

$$x_2 = \gamma(x_1 - vt_1) \tag{6}$$

Rewrite Eq. 4 $[x_2^2 + y_2^2 + z_2^2 - c^2t_2^2 = 0]$ using the above. Equate it $[= 0]$ with Eq. 3 $[x_1^2 + y_1^2 + z_1^2 - c^2t_1^2 = 0]$ to obtain

$$\begin{aligned} x_1^2 - c^2t_1^2 &= 0 = && x_2^2 - c^2t_2^2 \\ x_1^2 - c^2t_1^2 &= 0 = && \gamma^2(x_1 - vt_1)^2 - c^2t_2^2 \\ &= 0 = && \gamma^2(x_1^2 - 2x_1vt_1 + v^2t_1^2) - c^2t_2^2 \end{aligned}$$

Leads to

$$(1 - \gamma^2)x_1^2 + 2\gamma^2vx_1t_1 - \gamma^2v^2t_1^2 + c^2(t_2^2 - t_1^2) = 0. \tag{7}$$

It is inescapable that t_2 will depend on both t_1 and x_1.

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Finding the fix

Because t_2 must depend on both t_1 and x_1, assume a solution for t_2 of the form

$$t_2 = At_1 - Bvx_1. \tag{8}$$

Substituting this into Eq. 7 and collecting terms, we obtain

$$(1-\gamma^2+c^2v^2B^2)x_1^2+2(\gamma^2v-c^2vAB)x_1t_1+(c^2A^2-c^2-\gamma^2v^2)t_1^2 = 0. \tag{9}$$

Quadratic equation, which would have only two solutions of x_1. But we need something that works for all x_1! Must make coefficients be zero:

$$(1-\gamma^2+c^2v^2B^2) = 0 \quad \text{or} \quad c^2B^2 = \frac{\gamma^2-1}{v^2} \tag{10}$$

$$(\gamma^2 v - c^2vAB) = 0 \quad \text{or} \quad \gamma^4 = c^2A^2\,c^2B^2 \tag{11}$$

$$(c^2A^2 - c^2 - \gamma^2v^2) = 0 \quad \text{or} \quad c^2A^2 = \gamma^2v^2 + c^2. \tag{12}$$

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Simplifying

Again, we have $c^2B^2 = (\gamma^2 - 1)/v^2$ from Eq. 10, and $c^2A^2 = \gamma^2v^2 + c^2$ from Eq. 12, and $c^2A^2\, c^2B^2 = \gamma^4$ from Eq. 11. We can then substitute Eqs. 10 and 12 into Eq. 11 to get (using $\beta \equiv v/c$)

$$\begin{aligned}
\gamma^4 &= c^2A^2\, c^2B^2 = (\gamma^2v^2 + c^2)\frac{(\gamma^2 - 1)}{v^2} \\
&= (\gamma^2 + \frac{1}{\beta^2})(\gamma^2 - 1) = \gamma^4 + \frac{\gamma^2}{\beta^2} - \gamma^2 - \frac{1}{\beta^2} \\
\gamma^2(\frac{1}{\beta^2} - 1) &= \frac{1}{\beta^2} \\
\gamma^2 &= \frac{1}{\beta^2}\,\frac{1}{1/\beta^2 - 1} = \frac{1}{1 - \beta^2}
\end{aligned}$$

which finally gives

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \tag{13}$$

where $\beta \equiv v/c$.

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Simplifying II

Let's turn back to Eq. 10, and use $\beta \equiv v/c$:

$$\begin{aligned} c^2B^2 &= \frac{\gamma^2 - 1}{v^2} \\ c^4\beta^2B^2 &= \gamma^2 - 1 = \frac{1}{1-\beta^2} - 1 = \frac{1}{1-\beta^2} - \frac{1-\beta^2}{1-\beta^2} = \frac{\beta^2}{1-\beta^2} = \beta^2\gamma^2 \\ c^4B^2 &= \gamma^2 \\ B &= \frac{\gamma}{c^2} \end{aligned} \quad (14)$$

Let's also turn back to Eq. 12 of $c^2A^2 = \gamma^2 v^2 + c^2$:

$$A^2 = \gamma^2\beta^2 + 1 = \frac{\beta^2}{1-\beta^2} + \frac{1-\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2 \quad (15)$$

which gives $A = \gamma$ so that we can write our velocity transformation from Eq. 8 as

$$t_2 = At_1 - Bvx_1 = \gamma(t_1 - \frac{\beta}{c}x_1). \quad (16)$$

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Lorentz transformations

We have shown that the net transformation between coordinate systems is

$$x_2 = \gamma\,(x_1 - vt_1) \tag{17}$$
$$y_2 = y_1 \tag{18}$$
$$z_2 = z_1 \tag{19}$$
$$t_2 = \gamma\left(t_1 - \frac{\beta}{c}x_1\right) \tag{20}$$

with γ found from Eq. 13 to be

$$\gamma = \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}},$$

This transformation was first worked out by H. Lorentz in 1904, but in the assumption of a moving æther; as a result, Einstein gets the credit for getting the physics right. Important mathematical contributions later added by Poincaré and Minkowski.

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Binomial approximation

- Before we proceed further, let's remind ourselves of Taylor series approximations and the binomial approximation.
- In calculus, you learned that in the proximity of a particular point x_0 you can approximate a function $f(x)$ first by the value at that point, then a linear fit correction (first derivative df/dx), then a parabolic fit correction (second derivative d^2f/dx^2), and so on. That is, you get a Taylor series approximation:

$$\begin{aligned} f(x - x_0) \quad &\simeq \quad f(x)|_{x_0} + (x - x_0)\frac{df}{dx}|_{x_0} + \frac{(x-x_0)^2}{2!}\frac{d^2f}{dx^2}|_{x_0} \\ &\quad + \frac{(x-x_0)^3}{3!}\frac{d^3f}{dx^3}|_{x_0} + \ldots \end{aligned}$$

- Applying this to $(1+x)^n$ with $x \ll 1$ gives

$$(1+x)^n \simeq 1 + nx \qquad (21)$$

which is known as the *binomial approximation*.

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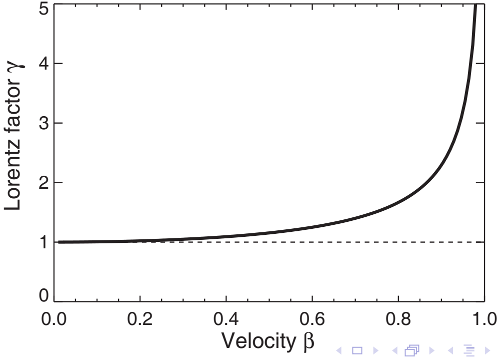
Lorentz factor γ

In the limit of $v \ll c$ or $\beta \ll 1$, we can use the binomial approximation to see how γ behaves:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = (1-\beta^2)^{-1/2} \simeq 1 + \frac{1}{2}\beta^2. \tag{22}$$

In the limit $\beta \to 0$, we have $\gamma \to 1$ and Eqs. 17 and 20 both revert to the Galilean results. This illustrates the all-important **correspondence principle** with classical physics.



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Inverse Lorentz transformations

Since observer at rest in S' or S_2 sees S or S_1 moving at $-v$, inverse transformations are

$$x_1 = \gamma\left(x_2 + vt_2\right) \quad (23)$$

$$y_1 = y_2 \quad (24)$$

$$z_1 = z_2 \quad (25)$$

$$t_1 = \gamma\left(t_2 + \frac{\beta}{c}x_2\right) \quad (26)$$

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Relativistic time dilation

- Fastest way to sense someone's clock ticks: speed of light.
- We are at rest in frame S_1; clock is in frame S_2, moving at v relative to us.
- Light pulses emitted in clock's frame at t_2 and then t'_2, so the period in frame S_2 is $T_2 = t'_2 - t_2$.
- Light pulses are emitted from same point on clock so $x_2 = x'_2$.
- To us (using Eq. 20):

$$t_1 = \gamma(t_2 + \frac{\beta}{c}x_2) \qquad \text{and} \qquad t'_1 = \gamma(t'_2 + \frac{\beta}{c}x'_2)$$

- Our time interval T_1:

$$T_1 = t'_1 - t_1 = \gamma\left((t'_2 - t_2) + \frac{\beta}{c}(x'_2 - x_2)\right) = \gamma T_2 \qquad (27)$$

We see time dilated by γ relative to the moving frame. This is often written as $t' = \gamma t_0$.

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The twin paradox

- At age 20, Sluggo stays home, while his twin Speedo travels to the Planet of the Apes, 10 light years from Earth, at $v = 0.5c$. Speedo then returns. Ignore crushing accelerations...
- To Sluggo, Speedo's journey took $t = x/v = 20$ years there, and 20 years back, or 40 years total.
- Sluggo's clock is dilated by γ relative to Speedo's; since $\gamma = 1/\sqrt{1-(0.5c/c)^2} = 1.15$, Speedo's clock ran for $40/1.15 = 34.6$ years.
- Upon Speedo's return, Sluggo is $20 + 40 = 60$ years old, while Speedo is $20 + 34.6 = 54.6$ years old!

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The twin paradox: experimental verification

Abstract, right? It's been tested! Atomic clocks have been flown around the world on jets. Time discrepancies in nanoseconds:
Eastward: measured -59 ± 10 nsec, predicted -40 ± 23 nsec
Westward: measured 273 ± 7 nsec, predicted 275 ± 21 nsec
See Hafele and Keating, *Science* **177**, 166 and 168 (1972). Note that the calculation involves both special relativity (plane going at a velocity relative to observer on earth's surface) and general relativity (acceleration from plane ascending and descending, and also from weaker gravitational attraction at altitude).

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Time travel?

There was a young lady named Bright,
Whose speed was far faster than light;
She set out one day
In a relative way,
And returned on the previous night

Attributed to Arthur Buller in P. Davies, "Wormholes and time machines," *Sky & Telescope* **83**, 20 (January 1992).

> *Of course, as a physics teacher I tell my students that faster-than-light travel is impossible, but that's just to crush their spirits.*

LaNelle Ohlhausen in the 1997-03 issue of the *Annals of Improbable Research*.

Calvin and Hobbes

Thanks to Sam Watterson. . .

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Length contraction

- We are at rest in frame S_1; measuring rod with ends separated by $L_2 = x_2' - x_2$ is in frame S_2 moving at v relative to us.
- We take a photograph of the rod at one instant of time, so for us $t_1 = t_1'$.
- We measure a distance $L_1 = x_1' - x_1$.
- Using Eq. 17, we can now relate the two lengths:

$$x_2 = \gamma\left(x_1 - vt_1\right) \qquad \text{and} \qquad x_2' = \gamma\left(x_1' - vt_1'\right)$$

- Relationship between perceived lengths is thus

$$L_2 = x_2' - x_2 = \gamma\Big((x_1' - x_1) - v(t_1' - t_1)\Big) = \gamma L_1 \qquad (28)$$

We see length contracted by $L_1 = (1/\gamma)L_2$. This is often written as $\ell' = \ell_0/\gamma$.