PHY 251 Fall 2009: homework problem set 5, due in the PHY 251 drop box in room A-129 by noon on Friday, Oct. 16.

1. A hydrogen atom is photoexcited from the $n = 1$ state to the $n = 20$ state. Calculate the photon energy needed to do this, and the photon energy needed to instead ionize the atom completely.
Answer: The energy needed to go from state n_i to state n_f is

$$\Delta E = E_{n_f} - E_{n_i} = -Z^2 E_0 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = Z^2 E_0 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right).$$

With hydrogen $(Z = 1)$ and $n_i = 1$ and $n_f = 20$, we have

$$\Delta E = 1^2 (13.60 \text{ eV}) \left(\frac{1}{1^2} - \frac{1}{20^2} \right) = (13.60 \text{ eV}) \frac{399}{400} = 13.57 \text{ eV}.$$

If instead $n_f \to \infty$ we have $1/n_f^2 \to 0$ and $\Delta E = 13.60$ eV.

2. The Lyman alpha line refers to the $n = 1$ to $n = 2$ transition in hydrogen, corresponding to the absorption of a photon. For light from a distant star, we see an absorption dip due to hydrogen in the periphery of the star at a wavelength of 421 nm. How fast is the galaxy moving away from us?
Answer: In our reference frame, the wavelength of the Lyman alpha line in hydrogen ($Z = 1$) is

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0(1/n_i^2 - 1/n_f^2)} = \frac{hc}{E_0(1/1^2 - 1/2^2)} = \frac{1240 \text{ eV} \cdot \text{nm}}{(13.60 \text{ eV})(3/4)} = 121 \text{ nm}.$$

Now the relativistic Doppler shift for a receding source ($\theta = 0$) is $\nu' = \nu_0/[\gamma(1 + \beta)]$. Let's define

$$x \equiv \frac{\nu_0}{\nu'} = \frac{\lambda'}{\lambda_0} = \frac{421 \text{ nm}}{121 \text{ nm}} = 3.48$$

and solve for β:

$$\begin{aligned}
x &= \gamma(1+\beta) = \frac{1+\beta}{\sqrt{1-\beta^2}} \\
x^2 &= \frac{(1+\beta)^2}{(1-\beta)(1+\beta)} = \frac{1+\beta}{1-\beta} \\
x^2 - x^2\beta &= 1 + \beta \\
x^2 - 1 &= \beta(x^2 + 1) \\
\beta &= \frac{x^2 - 1}{x^2 + 1} = \frac{(421/121)^2 - 1}{(421/121)^2 + 1} = 0.847
\end{aligned}$$

3. Calculate the wavelengths of the ground state to first excited state transition for hydrogen, deuterium, and tritium. Do this with enough precision to show the differences accurately.
Answer: The Bohr energy with a reduced mass of $m_r = m_e M/(m_e + M)$ is

$$E_n = -\frac{Z^2}{n^2} \frac{m_r e^4}{8h^2\epsilon_0^2} = -\frac{Z^2}{n^2} \frac{m_e e^4}{8h^2\epsilon_0^2} \frac{m_r}{m_e} = -\frac{Z^2}{n^2} E_0 \frac{m_r}{m_e}.$$

and find in turn that the wavelength corresponding to the transition from $n_i = 1$ to $n_f = 2$ with $Z = 1$ is given by

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_0(1/n_i^2 - 1/n_f^2)} = \frac{hc}{E_0(1/1^2 - 1/2^2)} = \frac{1240 \text{ eV} \cdot \text{nm}}{(13.60 \text{ eV})(3/4)} = 121 \text{ nm}$$

as we showed in problem 2. Now since $\lambda \propto 1/E_0$, and we replace E_0 with $E_0(m_r/m_e)$, we see that we need to scale the wavelengths by m_e/m_r. With nuclear masses M much larger than the electron mass m_e, we can find

$$\frac{m_e}{m_r} = \frac{m_e(m_e + M)}{m_e M} = \frac{m_e + M}{M} = (1 + \frac{m_e}{M})$$

and we can come up with numerical values for hydrogen, deuterium, and tritium using Appendix B on page A.2 of Serway and the atomic mass unit for an electron of $m_e = 5.486 \times 10^{-4}$ u:

$$\begin{array}{rl} \text{Hydrogen:} & (1 + \frac{m_e}{M}) = (1 + \frac{5.486 \times 10^{-4}}{1.007825}) = 1 + 0.00544 \\ & \text{so } \Delta\lambda = (121 \text{ nm}) \cdot (0.00544) = 0.066 \text{ nm} \\ \text{Deuterium:} & (1 + \frac{m_e}{M}) = (1 + \frac{5.486 \times 10^{-4}}{2.014102}) = 1 + 0.000272 \\ & \text{so } \Delta\lambda = (121 \text{ nm}) \cdot (0.00544) = 0.033 \text{ nm} \\ \text{Tritium:} & (1 + \frac{m_e}{M}) = (1 + \frac{5.486 \times 10^{-4}}{3.016049}) = 1 + 0.000182 \\ & \text{so } \Delta\lambda = (121 \text{ nm}) \cdot (0.00544) = 0.022 \text{ nm} \end{array}$$

So we can see that the difference in wavelength from hydrogen to deuterium is 0.066-0.033=0.033 nm, and from hydrogen to tritium is 0.066-0.022=0.044 nm.

4. Consider a muonic atom, where a muon (charge of an electron, but with a mass 207 times larger) is captured by a proton. What's the orbital radius, binding energy of the ground state, and wavelength of radiation required to go to the first excited state? Remember reduced mass...
Answer: In this case the reduced mass is

$$m_r = \frac{m_e M}{m_e + M} = \frac{(207 \cdot 0.511 \text{ MeV}/c^2) \cdot (938 \text{ MeV}/c^2)}{(207 \cdot 0.511 \text{ MeV}/c^2) + (938 \text{ MeV}/c^2)} = 95.1 \text{ MeV}/c^2$$

The energy of the ground state is

$$E_1 = -\frac{Z^2}{n^2} E_0 \frac{m_r}{m_e} = -\frac{1^2}{n^2}(13.60 \text{ eV}) \frac{95.1 \text{ MeV}/c^2}{0.511 \text{ MeV}/c^2} = 2531 \text{ eV}$$

while the next state has $1/2^2 = 1/4$ that energy or $2531/4 = 632.8$ eV. The energy difference is 2531-633=1898 eV and $\lambda = hc/\Delta E = (1240 \text{ eV} \cdot \text{nm})/(1898 \text{ eV}) = 0.65$ nm. The Bohr radius is

$$r_n = \frac{n^2}{Z} a_0 = \frac{n^2}{Z} \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} \Rightarrow \frac{n^2}{Z} a_0 \frac{m_e}{m_r}$$

when the reduced mass is applied. The radius of the ground state is then

$$r_1 = \frac{1^2}{1}(0.053\text{ nm})\frac{0.511\text{ keV}/c^2}{95.1\text{ MeV}/c^2} = 0.00028\text{ nm}.$$

5. Do Serway problem 4.37: Use Bohr's model of the hydrogen atom to show that when the atom makes a transition from the state n to the state $n-1$, the frequency of the emitted light is given by

$$f = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{(n-1)^2 n^2}\right]$$

where $k = 1/(4\pi\epsilon_0)$. Show that as $n \to \infty$, the preceeding expression varies as $1/n^3$ and reduces to the classical frequency one would expect the atom to emit. (*Hint:* To calculate the classical frequency, note that the frequency of revolution is $v/(2\pi r)$, where r is given by Equation 4.28.) This is an example of the correspondence principle, which requires that the classical and quantum models agree for large values of n.

Answer: I'll use ν for frequency, rather than f. The energy difference between states gives $\Delta E = h\nu$, so we have

$$h\nu = \Delta E = Z^2 E_0\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = Z^2\frac{m_e e^4}{8h^2\epsilon_0^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Let's look at the integer term with $n_i = n$ and $n_f = n-1$:

$$\frac{1}{n_f^2} - \frac{1}{n_i^2} = \frac{1}{(n-1)^2} - \frac{1}{n^2} = \frac{n^2}{(n-1)^2n^2} - \frac{(n-1)^2}{(n-1)^2n^2} = \frac{n^2-(n^2-2n+1)}{(n-1)^2n^2} = \frac{2n-1}{(n-1)^2n^2}.$$

Next, let's substitute $k = 1/(4\pi\epsilon_0)$ by multiplying by $1^2 = (4\pi\epsilon_0 k)^2$, and use $Z = 1$ for hydrogen:

$$\nu = \frac{1}{h}\frac{m_e e^4}{8h^2\epsilon_0^2}16\pi^2\epsilon_0^2 k^2\left[\frac{2n-1}{(n-1)^2n^2}\right] = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{(n-1)^2n^2}\right]$$

which is what we wanted to show. Now when $n \to \infty$, $n \gg 1$ so $(n-1) \to n$ and the term in brackets $[\,]$ becomes $2/n^3$ giving

$$\nu_{n\to\infty} \Rightarrow \frac{4\pi^2 m_e k^2 e^4}{n^3 h^3}.$$

Classically, one would expect the atom to emit radiation not due to the difference between energy states but due to the frequency of the electron's orbit around the atom (such as how a radio antenna emits, at the frequency of electron oscillation). This can be found from $f = v/2\pi r$ where for v we know that the centripetal force required for circular motion is provided by the electrostatic force:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \qquad \Rightarrow \qquad v = \left(\frac{ke^2}{mr}\right)^{1/2}$$

while for the orbital radius we use the Bohr result of $r = n^2\hbar^2/mke^2$. This gives

$$\begin{aligned} f &= \frac{v}{2\pi r} = \frac{1}{2\pi}\left(\frac{ke^2}{mr}\right)^{1/2}\frac{1}{r} = \frac{1}{2\pi}\,\frac{k^{1/2}e}{m^{1/2}}\,\frac{1}{r^{3/2}} = \frac{1}{2\pi}\,\frac{k^{1/2}e}{m^{1/2}}\,\frac{m^{3/2}k^{3/2}e^3}{n^3\hbar^3} \\ &= \frac{1}{2\pi}\,\frac{mk^2e^4}{n^3h^3}\,(2\pi)^3 = \frac{4\pi^2 mk^2e^4}{n^3h^3} \end{aligned}$$

so it all hangs together.

6. Do Serway problem 4.38: An electron with kinetic energy less than 100 eV collides head-on in an elastic collision with a massive mercury atom at rest. (a) If the electron reverses direction in a collision (like a ball hitting a wall), show that the electron loses only a tiny fraction of its initial kinetic energy, given by

$$\frac{\Delta K}{K} = \frac{4M}{m_e(1+M/m_e)^2}$$

where m_e is the electron mass and M is the mercury atom mass. (b) Using the accepted values for m_e and M, show that

$$\frac{\Delta K}{K} \simeq \frac{4m_e}{M}$$

and calculate the numerical value of $\Delta K/K$.

Answer: Because $K = 100$ eV is much less than $m_e c^2 = 511 \times 10^3$ eV, we're working in the classical limit. The electron's mass is $m_e = 5.486 \times 10^{-4}$ u where u is an atomic mass unit, while the mercury atom's mass is 200.59 u, so certainly $m_e \ll M$. Let's write the electron's kinetic energy after the collision as $K - \Delta K$, and write the mercury atom's kinetic energy after the collision as K_M. Now for an elastic collision the energy before is equal to the energy after, or

$$K = (K - \Delta K) + K_M \qquad \Rightarrow \qquad K_M = \Delta K.$$

Let's also write the speeds of the electron before and after as v and v', and of the mercury atom after as V. The conservation of energy expression then becomes

$$\frac{1}{2}m_e v^2 = \frac{1}{2}m_e v'^2 + \frac{1}{2}MV^2 \qquad \text{giving} \qquad v^2 = v'^2 + \frac{M}{m_e}V^2 = v'^2 + xV^2$$

with $x \equiv M/m_e$. The conservation of momentum equation is

$$m_e v = -m_e v' + MV \qquad \text{giving} \qquad v = -v' + \frac{M}{m_e}V = -v' + xV.$$

If we square the momentum expression and use it with the energy expression to eliminate v, we have

$$\begin{aligned} v'^2 + xV^2 &= (-v' + xV)^2 \\ v'^2 + xV^2 &= v'^2 - 2xv'V + x^2V^2 \\ 2xv'V &= (x^2 - x)V^2 \\ v' &= \frac{x-1}{2}V \end{aligned}$$

Substituting this into the conservation of energy equation gives

$$\begin{aligned} v^2 &= v'^2 + xV^2 = \frac{(x-1)^2}{4}V^2 + xV^2 = \frac{x^2 - 2x + 1 + 4x}{4}V^2 \\ &= \frac{x^2+2x+1}{4}V^2 = \frac{(x+1)^2}{2^2}V^2 \end{aligned}$$

or $V = 2v/(x+1)$. The conservation of energy expression is then

$$\begin{aligned} v^2 &= v'^2 + xV^2 = v'^2 + x\frac{2^2}{(x+1)^2}v^2 \\ v'^2 &= v^2 - \frac{4x}{(x+1)^2}v^2 \end{aligned}$$

from which we get ΔK as

$$\Delta K = \frac{1}{2}m_e v^2 - \frac{1}{2}mv'^2 = \frac{1}{2}m_e\left[v^2 - v^2 + \frac{4x}{(x+1)^2}v^2\right] = \frac{1}{2}m_e v^2\frac{4x}{(x+1)^2} = K\frac{4x}{(x+1)^2}$$

which leads to

$$\frac{\Delta K}{K} = \frac{4x}{(x+1)^2} = \frac{4M}{m_e(1+M/m_e)^2}.$$

Now if $M \gg m_e$ we can ignore the 1 in $(1 + M/m_e)$ and approximate our result as

$$\frac{\Delta K}{K} \simeq \frac{4M}{m_e M^2/m_e^2} = 4\frac{m_e}{M} = 4\frac{5.486\times 10^{-4}}{200.59} = 1.1\times 10^{-5}$$

so that the change in the electron's kinetic energy is pretty small.

Note that we could have gotten the final approximate result somewhat more directly. The conservation of momentum along with the classical result that $E_k = p^2/2m$ or $p = \sqrt{2mE_k}$ gives

$$\sqrt{2m_eK} = \sqrt{2MK_M} - \sqrt{2m_e(K-\Delta K)}$$

Now the simple way to proceed is to go directly to a lowest-order term expansion:

$$\begin{aligned} 1 &= \sqrt{\frac{M}{m_e}\frac{\Delta K}{K}} - \sqrt{1 - \frac{\Delta K}{K}} \qquad \text{(since } K_M = \Delta K\text{)} \\ 1 &= \sqrt{\frac{M}{m_e}\frac{\Delta K}{K}} - \left(1 - \frac{1}{2}\frac{\Delta K}{K}\right) \\ 2\left(1 + \frac{1}{4}\frac{\Delta K}{K}\right) &= \sqrt{\frac{M}{m_e}\frac{\Delta K}{K}} \qquad \text{(binomial approx.)} \\ 4\left(1 + \frac{1}{2}\frac{\Delta K}{K}\right) &= \frac{M}{m_e}\frac{\Delta K}{K} \qquad \text{(binomial after squaring)} \\ 4 &= \frac{\Delta K}{K}\left(\frac{M}{m_e} - 2\right) \qquad \text{(and } \frac{M}{m_e} \gg 2 \text{ so drop 2)} \\ \text{giving} \qquad \frac{\Delta K}{K} &\simeq 4\frac{m_e}{M}. \end{aligned}$$

7. Calculate the de Broglie wavelength of a 5 eV electron, a 100 keV electron, a 1 TeV proton, and a Volkswagen Beetle covered with daisy stickers traveling at 20 m/s.
Answer: For the 5 eV electron classical physics suffices, so from $E_k = p^2/2m$ and $\lambda = h/p$ we get

$$\lambda = \frac{hc}{\sqrt{2E_k mc^2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2 \cdot (5 \text{ eV}) \cdot (511 \times 10^3 \text{ eV})}} = 0.55 \text{ nm}.$$

For the Volkswagen New Beetle, a check on the Volkswagen web site tells us that the curb weight is 1345 kg so

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ m}}{(1345 \text{ kg})(20 \text{ m/s})} = 2.5 \times 10^{-38} \text{ meters}$$

For a relativistic particle, we have $E_k = (\gamma - 1)mc^2$ or $\gamma = 1 + E_k/mc^2$. Once we know γ we have $\beta = \sqrt{1 - 1/\gamma^2}$, we have $p = \gamma\beta mc$, and we have $\lambda = h/p = hc/pc$:

$$\begin{aligned}
\gamma &= 1 + \frac{E_k}{mc^2} = 1 + \frac{[10^5, 10^{12}] \text{ eV}}{[511 \times 10^3, 939 \times 10^6] \text{ eV}} = [1.196, 1067] \\
\beta &= \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/([1.196, 1067])^2} = [0.548, 1] \\
p &= \gamma\beta mc^2/c = [1.196, 1067] \cdot [0.548, 1] \cdot [511 \times 10^3, 938 \times 10^6] = [335 \times 10^3, 1.00 \times 10^{12}] \text{ eV/c} \\
\lambda &= \frac{hc}{pc} = \frac{1240 \text{ ev} \cdot \text{nm}}{[335 \times 10^3, 1.00 \times 10^{12}] \text{ eV}} = [3.7 \times 10^{-3}, 1.2 \times 10^{-9}] \text{ nm}.
\end{aligned}$$

8. Serway problem 5.14: Show that the formula for low-energy electron diffraction (LEED) , when electrons are incident perpendicular to a crystal surface, may be written as

$$\sin\phi = \frac{nhc}{d\sqrt{2m_e c^2 K}}$$

where n is the order of the maximum, d is the atomic spacing, m_e is the electron mass, K is the electron's kinetic energy, and ϕ is the angle between the incident and diffracted beams. (b) Calculate the atomic spacing in a crystal that has consecutive diffraction maxima at $\phi = 24.1°$ and $\phi = 54.9°$ for 100 eV electrons.
Answer: These are low-energy electrons, so they're in the classical limit and the kinetic energy is $E_k = p^2/2m$ giving $p = \sqrt{2mE_k}$. The de Broglie wavelength is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2m_e c^2 E_k}}.$$

If we say that the crystal planes form a simple grating at the top surface (because the electrons don't travel very far in the material), and the electron beam scatters off of that grating, then we have a simple, planar grating equation (as opposed to a volume or Bragg grating equation of $2d\sin\theta = n\lambda$). The simple grating equation is

$$\sin\theta = \frac{n\lambda}{d} = \frac{nhc}{d\sqrt{2(m_e c^2)E_k}}.$$

Well, let's move on to part B. If we have two angles for successive peaks n and $n+1$, we can write

$$\sin\theta_{n+1} - \sin\theta_n = (n+1)\frac{hc}{d\sqrt{2(mc^2)E_k}} - n\frac{hc}{d\sqrt{2(mc^2)E_k}} = \frac{hc}{d\sqrt{2(mc^2)E_k}}$$

We can then solve for the atomic spacing d:

$$\begin{aligned} d &= \frac{hc}{(\sin\theta_{n+1} - \sin\theta_n)\sqrt{2(mc^2)E_k}} \\ &= \frac{1240 \text{ eV} \cdot \text{nm}}{(\sin 54.9^\circ - \sin 24.1^\circ)\sqrt{2 \cdot 511 \times 10^3 \text{ eV} \cdot 100 \text{ eV}}} = (0.30 \text{ nm}) \end{aligned}$$

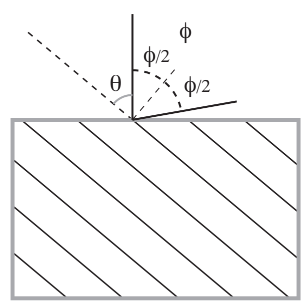
or $d = 3$ Å which is a reasonable inter-atomic distance or lattice spacing in solids. By the way, we can also infer the diffraction orders n from realizing that $\sin\theta = (\lambda/d)n$ so if we look at the ratio of the two angles of

$$\frac{\sin\theta_A}{\sin\theta_B} = \frac{(\lambda/d)n_A}{(\lambda d)n_B} = \frac{n_A}{n_B}$$

which in this case is $\sin 54.9^\circ / \sin 24.1^\circ = 0.818/.408 \simeq 2/1$ so we can guess that $n_A = 2$ and $n_B = 1$.

It's worth comparing the above result with a volume or Bragg grating result (see right). In this case, the net deflection angle is ϕ, and the Bragg angle θ is given by $\theta = \pi/2 - \phi/2$. Therefore we can write Bragg's law as

$$2d\sin\theta = 2d\sin(\pi/2 - \phi/2) = 2d\cos(\phi/2) = n\lambda.$$



We then have

$$\begin{aligned} \lambda &= \frac{2d\cos(\phi/2)}{n} = \frac{hc}{\sqrt{2m_e c^2 E_k}} \\ \text{or} \qquad 2\cos(\phi/2) &= \frac{nhc}{d\sqrt{2m_e c^2 E_k}}. \end{aligned}$$

The moral of the story is that one gets significantly different results from scattering from a simple grating on a surface, or scattering from a volume grating.

9. Imagine making an "atom" where a nucleus of 20 neutrons has in its orbit a single neutron, where gravitational attraction rather than electrostatic attraction holds the "atom" together. Calculate the energy and radius of the ground state.
Answer: The general idea here is to replace the electrostatic force with gravity. Let's say that the "nucleus" has a mass of Zm with $Z = 20$ and $m = 1.675 \times 10^{-27}$ kg, while the "electron" has a mass of m. The centripetal force required to keep the "electron" in uniform

circular motion is provided by the gravitational force. We rearrange to obtain an expression in angular momentum ℓ, and assume $\ell = n\hbar$:

$$m\frac{v^2}{r} = G\frac{m \cdot Zm}{r^2} \qquad \Rightarrow \qquad \ell^2 = (rmv)^2 = GZm^3 r = (n\hbar)^2$$

which in turn gives

$$\begin{aligned} r &= \frac{n^2\hbar^2}{GZm^3} = n^2\frac{(1.055\times10^{-34})^2}{(6.67\times10^{-11})(20)(1.675\times10^{-27})^3} \\ &= n^2\frac{(1.055)^2}{6.67\cdot 20\cdot 1.675^3}\,10^{-2\cdot34+11+3\cdot27} = n^2(1.77\times10^{-3})\,10^{24} = n^2\,1.77\times10^{21}\text{ meters.}\end{aligned}$$

Of course we used SI units for all constants so that the calculation result is in SI units. For perspective, this radius is about the same as the diameter of our galaxy (the Milky Way)! Next, since $mvr = \ell\hbar$, we can calculate the kinetic energy of the “electron”:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}r\left(\frac{mv^2}{r}\right) = \frac{1}{2}r\left(G\frac{Zm^2}{r^2}\right) = G\frac{Zm^2}{2r}.$$

The gravitational potential is $U = -GZm^2/r$, so that the total energy is

$$E = E_k + U = G\frac{Zm^2}{2r} - G\frac{Zm^2}{r} = -G\frac{Zm^2}{2r}.$$

When we substitute the radius found above into this expression, we find

$$E_n = -G\frac{Zm^2}{2}\,\frac{GZm^3}{\hbar^2}\frac{1}{n^2} = -\frac{Z^2}{n^2}\,\frac{G^2m^5}{2\hbar^2}$$

where the numerical value of the constants is

$$\begin{aligned}\frac{G^2m^5}{2\hbar^2} &= \frac{(6.67\times10^{-11})^2(1.675\times10^{-27})^5}{2(1.055\times10^{-34})^2} \\ &= \frac{6.67^2\cdot1.675^5}{2\cdot1.055^2}\,10^{-2\cdot11-5\cdot27+2\cdot34} = (2.64\times10^2)\,10^{-89} = 2.64\times10^{-87}\text{ Joules.}\end{aligned}$$

The energy of the ground state is then

$$E_1 = -\frac{Z^2}{n^2}\,\frac{G^2m^5}{2\hbar^2} = -\frac{20^2}{1^2}\,(2.64\times10^{-87}\text{ Joules}) = 1.06\times10^3\cdot10^{-87}\text{ Juoles}$$

or $E_1 = 1.06\times10^{-84}$ Joules, or 6.62×10^{-67} eV. What a different world it would be!!!