PHY 251 Fall 2009: homework problem set 1, due PHY 251 drop box in room A-129 by noon on Friday, Sep. 11.

1. Show that if one uses the Galilean relativity transformation of

$$\begin{aligned} x_2 &= x_1 - vt_1 \\ y_1 &= y_2 \\ z_1 &= z_2 \\ t_1 &= t_2 \end{aligned} \quad (1)$$

   and the relationships for expansion of light spheres of

$$x_1^2 + y_1^2 + z_1^2 - c^2 t_1^2 = 0 \quad (2)$$
$$x_2^2 + y_2^2 + z_2^2 - c^2 t_2^2 = 0. \quad (3)$$

   that you get a non-general and inconsistent-with-classical-physics result.

2. Show how classical relativity affects speed in an airplane flight with either headwinds and tailwinds, or crosswinds. Let's say that the wind is blowing east west at a velocity of $v = 10$ m/s, and that you're in an airplane that travels at a speed of $c = 250$ m/s. Consider a round-trip journey where you fly 300 km out and back as measured on the ground, going east then west. Then consider the case where you go straight south then north as measured from the ground (what compass heading should you fly the plane at to accomplish this?). What's the time difference between the two trips? Work out an algebraic answer approximated to lowest order in $\beta = v/c$ before you plug in numbers.

3. How fast must a meter stick be moving past you for it to appear to you to be only 0.6 m long?

4. It is said that "time flies when you're having fun." Some people having fun go past you at $v = 3 \times 10^6$ m/s. You see one of them laugh for 20 seconds straight as observed in your frame. How much different is the duration of their laugh in their frame?

5. While staring out the window, you are somewhat astonished to see a spear go flying past, and you snap a photograph. From your photo you see that it is 1 meter long and tilted up at an angle of 20° from the x direction, and it goes flying past at $v_x = 1 \times 10^8$ m/sec and $v_y = 0$. What is length, and tilt angle, of the spear in its own inertial reference frame?

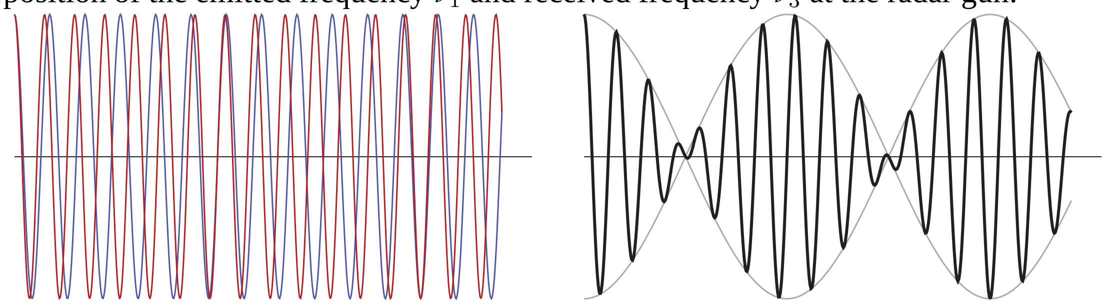
6. A cop pulls you over for running a red light ($\lambda = 650$ nm). Thinking that you're a clever physics student, you go to court and point out that due to the relativistic Doppler shift it looked like a green ($\lambda = 550$ nm) light to you, smiling smugly at the judge. However, the judge immediately slams you with a speeding ticket instead. How fast does the judge know you were going?

7. For $v \ll c$, show that the relativistic Doppler shift leads to relative frequency and wavelength shifts of

$$\frac{\Delta\nu}{\nu} \simeq -\frac{v}{c} \qquad \text{and} \qquad \frac{\Delta\lambda}{\lambda} \simeq \frac{v}{c}.$$

Apply this to find the velocity at which a galaxy must be receding if we see a spectral feature at $\lambda = 400$ nm redshifted by 15 nm.

8. A radar speed gun works by sending out a radar wave at a frequency ν_1. A moving object sees this frequency shifted to ν_2, and reflects that radar signal back to the radar gun. From the radar gun's point of view, we have a moving emitter at frequency ν_2 which leads to a frequency ν_3 seen by the radar gun. We then get a beat frequency $\nu_b = \nu_3 - \nu_1$ from the superposition of the emitted frequency ν_1 and received frequency ν_3 at the radar gun:



Show that, to lowest order in (v/c), the beat frequency is given by $\nu_b = 2(v/c)\nu_1$, and calculate the beat frequency the radar gun sees when measuring an object moving at $v = 30$ m/sec using a radar frequency of 10.0 GHz.

9. As a still-lurid 80-year old in 2061, Paris Hilton drives a Hummer H34 spaceship which is 15 m long. Al Gore, long since retired but still kicking around, drives a Honda CivicsLesson spaceship which is only 3 m long. Paris and Al both fly past you while you're standing on earth, and it looks to you like they're both driving spaceships of the same length. You know that Al drives at, but not even a smidgen above, the posted speed limit of $v = 0.55c$. How fast is Paris going relative to you on earth? Relative to Al?

10. The nearest star to earth is Proxima Centauri, at 4.2 light years distance. If you were instantaneously able to accelerate to $\beta = 0.1$, how long would the journey appear to take to you in your spaceship? Next, using classical physics, estimate how long it would take for you to accelerate to $\beta = 0.1$ if your acceleration was limited to 2.0g.