Physics 251 exam 3, May 18, 2009. You may use a calculator, and the equation sheet that I have provided.
Order of atoms in the periodic table: H, He, Li, Be, B, C, N, O, F, Ne, Na, Mg... And here are some isotopic masses you might find to be of use:

$^{4}_{2}$He:	4.002 603	$^{142}_{54}$Xe:	141.929 701	$^{143}_{55}$Cs:	142.927 330	$^{144}_{55}$Cs:	143.932 030
$^{145}_{55}$Cs:	144.935 390	$^{143}_{56}$Ba:	142.920 617	$^{144}_{56}$Ba:	143.922 940	$^{145}_{56}$Ba:	144.926 920
$^{143}_{57}$La:	142.916 059	$^{144}_{57}$La:	143.919 590	$^{145}_{57}$La:	144.921 641	$^{235}_{92}$U:	235.043 923
$^{236}_{92}$U:	236.045 562	$^{237}_{92}$U:	237.048 724	$^{238}_{92}$U:	238.050 783	$^{239}_{92}$U:	239.054 288
$^{237}_{93}$Np:	237.048 167	$^{238}_{93}$Np:	238.050 941	$^{239}_{93}$Np:	239.052 931	$^{239}_{94}$Pu:	239.052 157
$^{238}_{95}$Am:	235.048 032	$^{239}_{95}$Am:	239.053 018	$^{240}_{95}$Am:	240.055 288	$^{239}_{96}$Cm:	239.054 951
$^{240}_{96}$Cm:	240.055 519	$^{241}_{96}$Cm:	241.057 647				

1. Ultraviolet light with $\lambda = 150$ nm is shined on a metal, and a stopping potential of 3.0 Volts is sufficient to make the detected current go to zero in a photoelectric effect experiment. What's the work function of the metal?
   Answer: The photon energy hc/λ goes towards overcoming the work function φ of the metal and giving the electron a kinetic energy E_k which is equal to the stopping potential times electron charge or qV. That is,

$$\varphi = \frac{hc}{\lambda} - E_k = \frac{1240 \text{ eV} \cdot \text{nm}}{150 \text{ nm}} - (3.0 \text{ eV}) = 8.27 - 3.0 = 3.27 \text{ eV}$$

2. An electron is accelerated through a 300 kV potential. What's its velocity? Its de Broglie wavelength? What strength of magnetic field is required to make it curve with a radius of 0.5 meters?
   Answer: The electron acquires a kinetic energy of $qV = 300$ keV, so we must account for relativity as its rest mass is only 511 keV/c^2. Therefore we have

$$qV = (\gamma - 1)mc^2 \qquad \text{or} \qquad \gamma = 1 + \frac{qV}{mc^2} = 1 + \frac{300 \times 10^3 \text{ eV}}{511 \times 10^3 \text{ eV}} = 1.58$$

   and its velocity can be found from

$$\begin{aligned} \gamma &= 1/\sqrt{1-\beta^2} \\ 1-\beta^2 &= 1/\gamma^2 \\ \beta &= \sqrt{1-1/\gamma^2} = \sqrt{1-1/(1.58^2)} = 0.774 \end{aligned}$$

   or $v = 0.774 \cdot 3 \times 10^8$ m/sec or 2.3×10^8 m/sec. Next, the de Broglie wavelength is $\lambda = h/p$ but since $p = \gamma m_0 v$ and $v = \beta c$ we have

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m \beta c} = \frac{hc}{\gamma mc^2 \beta} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.58 \cdot 511 \times 10^3 \text{ eV} \cdot 0.774} = 0.00198 \text{ nm}.$$

Finally, the radius of curvature can be found from $qvB = \gamma mv^2/r$ giving

$$B = \frac{\gamma mv^2}{qvr} = \frac{\gamma mc\beta}{qr} = \frac{1.58 \cdot (9.11 \times 10^{-31}\text{ kg}) \cdot (3.00 \times 10^8\text{ m/s}) \cdot (0.774)}{(1.602 \times 10^{-19}\text{ C}) \cdot (0.5\text{ m})}$$

or $B = 0.0042$ Tesla.

3. A tritium nucleus (hydrogen plus two neutrons; assume that a neutron mass equals a proton mass) captures a muon (mass 105.7 MeV/c^2) to briefly form an exotic atom. Calculate the energy and Bohr model orbital radius of the first excited state (not the ground state).
Answer: The reduced mass is

$$m_r = \frac{3m_p m_\mu}{3m_p + m_\mu} = \frac{3 \cdot 939 \times 10^6 \cdot 105.7 \times 10^6}{3 \cdot 939 \times 10^6 + 105.7 \times 10^6} = 102\text{ MeV/c}^2.$$

The energy of the first excited state ($n = 2$) is the Bohr model energy times m_r/m_e to account for the difference relative to the electron mass (because $E \propto m$). We then have

$$E_2 = -\frac{1^2}{2^2}\frac{102\text{ MeV}}{0.511\text{ MeV}}\, 13.6\text{ eV} = 679\text{ eV}.$$

Because $r \propto 1/m$, we multiply the Bohr model radius by m_e/m_r, giving

$$r_2 = \frac{2^2}{1}\frac{0.511\text{ MeV}}{102\text{ MeV}}\, 0.053\text{ nm} = 0.0011\text{ nm}$$

4. Light from the $2p \to 1s$ transition in hydrogen in atoms in a distant galaxy is observed at a wavelength of $\lambda = 300$ nm. What's the velocity of the galaxy relative to earth?
Answer: The photon energy of the transition is given by

$$\Delta E = E_3 - E_2 = -E_0 Z^2 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) = -(13.6\text{ eV})(1^2)\left(-\frac{3}{4}\right) = 10.2\text{ eV}$$

or $\lambda = hc/\Delta E = (1240\text{ eV} \cdot \text{nm})/(10.2\text{ eV}) = 122$ nm. If it's instead observed at a wavelength of $\lambda = 300$ nm, we have $\nu/\nu_0 = \lambda_0/\lambda = 122/300$. The relativistic Doppler shift for a receding source involves $\theta = 0$ in

$$\begin{aligned}
\nu &= \nu_0 \frac{1}{\gamma[1+\beta\cos\theta]} = \nu_0 \frac{1}{\gamma[1+\beta]} = \nu_0 \frac{\sqrt{1-\beta^2}}{1+\beta} \\
&= \nu_0 \frac{\sqrt{(1-\beta)(1+\beta)}}{\sqrt{(1+\beta)(1+\beta)}} = \nu_0 \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \\
(\frac{\nu}{\nu_0})^2 &= \frac{1-\beta}{1+\beta} \\
(\frac{\nu}{\nu_0})^2 + \beta(\frac{\nu}{\nu_0})^2 &= 1-\beta \\
\beta\left(1 + (\frac{\nu}{nu_0})^2\right) &= 1 - (\frac{\nu}{\nu_0})^2 \\
\beta &= \frac{1-(\nu/\nu_0)^2}{1+(\nu/\nu_0)^2} = \frac{1-(122/300)^2}{1+(122/300)^2}
\end{aligned}$$

or $\beta = 0.72$, giving $v = \beta c = (0.72) \cdot 2.99 \times 10^8$ m/sec or 2.2×10^8 m/sec.

5. While running at $v = 0.2c$ into a Manhattan nightclub, Plaxico Burress trips so that his gun flies out of his sweatpants' waistband with a velocity of $v = 0.3c$ away from Plaxico, and then the gun discharges a bullet back towards towards Plaxico's leg. If the lightweight bullet leaves the very heavy gun at a velocity of $v = 0.5c$ from the gun's point of view, what's the speed of the bullet as seen by Plaxico before it hits his leg? *And a word to the wise: don't bring a gun to a nightclub, and for sure don't tuck it into your sweatpants' waistband!*
Answer: Because the gun is heavy and the bullet is light, we can neglect issues such as a momentum kick to gun. Thus we have a standard problem of relativistic velocities. From the gun's point of view, the bullet is flying out with $v_{1,x} = 0.5c$. To go to Plaxico's point of view, we have a frame shift velocity (in the same direction) of $v = 0.3c$. As a result, Plaxico sees a bullet velocity of

$$v_{2,x} = \frac{v_{1,x} - v}{1 - v v_{1,x}/c^2} = \frac{0.5c - 0.3c}{1 - (0.3c)(0.5c)/c^2} = \frac{0.2c}{1 - 0.15} = 0.235c$$

6. In Dan Brown's next novel, he will postulate that a secretive society has been jealously guarding a 1 kg sphere of ^{239}Pu that was somehow produced by Moses 3500 years ago. Although accuracy has not gotten in the way of entertainment before, you are hired to calculate the alpha decay energy output of this sphere as measured today so that Robert Langdon's beautiful, mysterious sidekick can authoritatively say if the sphere is that old or not. Knowing that ^{239}Pu alpha decays with a half-life of 2.41×10^4 years, what's your answer in Watts? *And what movie did I see on Saturday, coincidentally with K.C. from this class?*
Answer: The reaction is ${}^{239}_{94}\text{Pu} \rightarrow^4_2 \alpha +^{235}_{92} \text{U}$. The energy released per decay is

$$Q = m_{\text{Pu}} - m_\alpha - m_{\text{U}} = 239.052\,157 - 4.002\,603 - 235.043\,923 = 0.005\,631 \text{ u}$$

or $(0.00563 \text{ u}) \cdot (931.494 \text{ MeV/u}) = 5.29$ MeV. Now we will want to know the number of seconds per year:

$$(60 \text{ s/m}) \cdot (60 \text{ m/h}) \cdot (24 \text{ h/d}) \cdot (365 \text{ d/y}) = 3.15 \times 10^7 \text{ s/y}.$$

The activity is given by $R = \lambda N$ or, since $t_{1/2} = \ln 2/\lambda$, the activity of a new 1 kg mass of ^{239}Pu is

$$\begin{aligned} R &= \lambda N = \frac{\ln 2}{t_{1/2}} \cdot \frac{m\, N_A}{A} \\ &= \frac{\log 2}{2.41 \times 10^4 \text{ y}} \cdot \frac{1 \text{ y}}{3.15 \times 10^7 \text{ s}} \cdot \frac{1 \times 10^3 \text{ g}}{239.052 \text{ g/mole}} \cdot \frac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}} \\ &= 2.30 \times 10^{12} \frac{\text{decays}}{\text{sec}} \end{aligned}$$

The power from the fresh 1 kg mass is then

$$\frac{5.29 \times 10^6 \text{ eV}}{\text{decay}} \cdot 2.30 \times 10^{12} \frac{\text{decays}}{\text{sec}} \cdot 1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}} = 1.95 \text{ W}$$

whereas if it is 3500 years old its activity has decayed according to $\exp[-\lambda t]$ or

$$\exp\Big[-\frac{\ln 2}{t_{1/2}}t\Big] = \exp\Big[-0.693 \cdot \frac{3.5 \times 10^3 \text{ years}}{24.1 \times 10^3 \text{ years}}\Big] = 0.904$$

so the power from a 3500 year old 1 kg mass is only 1.76 W.

7. A particle is trapped in an infinite one-dimensional well of width L. If the particle is in its ground state, calculate the probability of finding the particle between $x = 0$ and $x = L/3$. Note: $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$.
   Answer: The wave function is $\psi = \sqrt{\frac{2}{L}} \sin n\pi\frac{x}{L}$, so the probability is

$$\begin{aligned}
\int_0^{L/3} \psi^*\psi\, dx &= \int_0^{L/3} \frac{2}{L}\sin^2 n\pi\frac{x}{L}\, dx \\
&= \frac{2}{L}\int_0^{L/3}\frac{1}{2}\, dx - \frac{2}{L}\int_0^{L/3}\frac{1}{2}\cos\frac{2\pi}{L}x\, dx \\
&= \frac{2}{L}\frac{1}{2}x|_0^{L/3} - \frac{2}{L}\frac{L}{2\pi}\sin\frac{2\pi}{L}x|_0^{L/3} \\
&= \frac{2}{L}\frac{1}{2}[\frac{L}{3} - 0] - \frac{2}{L}\frac{L}{2\pi}[\sin\frac{2}{3}\pi - \sin 0] \\
&= \frac{1}{3} - \frac{1}{2\pi}\sin\frac{2}{3}\pi = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} = 0.196
\end{aligned}$$

8. As a result of Compton scattering with a 100 keV incident photon, a photon is scattered at $60°$. What's the energy of the scattered photon, and the energy and approximate momentum of the scattered electron?
   Answer: The wavelength of the scattered photon is found from

$$\begin{aligned}
\lambda_s - \lambda_0 &= \frac{h}{mc}(1 - \cos\theta) \qquad \text{or} \qquad \lambda_s = \lambda_0 + \frac{h}{mc}(1 - \cos\theta) \\
\lambda_s &= \frac{hc}{100 \text{ keV}} + \frac{hc}{mc^2}(1 - \cos 60°) = \frac{1240 \text{ eV}\cdot\text{nm}}{100 \times 10^3 \text{ eV}} + \frac{1240 \text{ eV}\cdot\text{nm}}{511 \times 10^3 \text{ eV}}(1 - \cos 60°) \\
&= 0.01361 \text{ nm}
\end{aligned}$$

which corresponds to a photon energy of hc/λ of 91.09 keV. Therefore the electron has a kinetic energy of 100-91.09=8.91 keV, giving $\gamma = 1 + E_k/mc^2 = 1 + 8.91/511 = 1.017$ which we'll call $1 + x$ with $x = 0.017$. If we wanted to be fancy we could use this to find the velocity using

$$\beta = \sqrt{1 - 1/\gamma^2} = \Big(1 - 1/(1+x)^2\Big)^{1/2} \simeq \Big(1 - (1-2x)\Big)^{1/2} = (2x)^{1/2} = (2\cdot 0.017)^{1/2} = 0.184$$

though of course we could also brute-force it using $\beta = \sqrt{1 - 1/\gamma^2}$ which also gives $\beta = 0.184$. We can then find momentum from

$$p = \gamma mv = \gamma mc^2\beta/c = (1.017)(511 \times 10^3 \text{ keV})(0.184)/c = 95.6 \text{ keV}/c$$

Of course the classical physics result can be found from $E_k = p^2/(2m)$ or

$$p = \sqrt{2mE_k} = \sqrt{2mc^2\,E_k}/c = \sqrt{2 \cdot (511 \times 10^3\ \text{keV}) \cdot (8.91\ \text{keV})}/c = 95.4\ \text{keV}/c$$

which is close.

9. Using only two pages of your blue book, describe and sketch what's involved in designing and producing a nuclear bomb. *Extra credit: MacGyver one together using materials available in the exam room, within the exam time. For those who have studied physics more than TV, substitute "Improvise the construction of one" for "MacGyver one together."*
Answer: Should mention isotope enrichment of uranium, plutonium production, critical mass, gun-type assembly, implosion-type assembly.

10. With the restriction that you can use only two pages of your blue book, write for me a short summary of this course. Include names, dates, and equations. Of course you'll only be able to describe a few highlights, but show how these most important highlights were historically and conceptually linked.
Answer: Should include Einstein and relativity, Planck and Einstein and photons, Rutherford and small dense nucleus, the Bohr model, de Broglie, Schrödinger and wave equations, and then perhaps something on the nucleus.