Physics 251 exam 1, March 4, 2009. You may use a calculator, and the equation sheet that I have provided.

1. Blubbo, a rather rotund, 2-meter-diameter superhero, has an amazing ability to heal as long as a spear is fully contained within him for a moment in time from his perspective, rather than sticking out of him front and back. Bruto throws a spear (it measures 3 meter long in its own inertial frame) at Blubbo. How fast does the spear need to traveling for Blubbo to survive the attack?
   Answer: From Blubbo's perspective, the spear needs to be relativistically contracted from 3 meters to 2 meters, or (2 meters)=(3 meters)/γ or $\gamma = 3/2$, giving

   $$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 4/9} = \sqrt{5/9} = 0.745$$

   or $v = 0.745c = 2.24 \times 10^8$ m/sec.

2. While rushing to arrive nice and early for your 8:20 am recitation section so that you make your professor exceedingly happy, you get pulled over by a police officer for running a red ($\lambda = 600$ nm) light. You tell the police officer that it looked green ($\lambda = 500$ nm) to you. The police officer responds by adding a speeding ticket to your list of offences. What speed does the police officer write down on the ticket?
   Answer: Let's define $A = \lambda_0/\lambda = \nu/\nu_0$. The relativistic Doppler shift for an approaching source involves $\theta = 180°$ is

   $$\begin{aligned}
   \nu &= \nu_0 \frac{1}{\gamma[1 + \beta\cos\theta]} = \nu_0 \frac{1}{\gamma[1 - \beta]} = \nu_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta} \\
   A &= \frac{\sqrt{(1 - \beta)(1 + \beta)}}{1 - \beta} = \frac{\sqrt{1 + \beta}}{\sqrt{1 - \beta}} \\
   A^2 &= \frac{1 + \beta}{1 - \beta} \\
   A^2 - \beta A^2 &= 1 + \beta \\
   \beta &= \frac{A^2 - 1}{A^2 + 1} = \frac{(600/500)^2 - 1}{(600/500)^2 + 1} = 0.180
   \end{aligned}$$

   giving $v = \beta c = (0.180) \cdot 2.99 \times 10^8$ m/sec or 5.38×10^7 m/sec.

3. The stopping potential in a photoelectric effect experiment is 1.2 volts when $\lambda = 300$ nm light is used. What's the kinetic energy of the electrons produced when $\lambda = 250$ nm light is used and no retarding voltage is applied?
   Answer: The stopping potential equals the kinetic energy of the electrons when no retarding voltage is applied. Because $E_k = hc/\lambda - \varphi$, from the first case we have

   $$\varphi = \frac{hc}{\lambda} - E_k = \frac{1240 \text{ eV} \cdot \text{nm}}{300 \text{ nm}} - (1.2 \text{ eV}) = 2.93 \text{ eV}.$$

If we change the illuminating wavelength to 250 nm, we have

$$E_k = \frac{hc}{\lambda} - \varphi = \frac{1240 \text{ eV} \cdot \text{nm}}{250 \text{ nm}} - (2.93 \text{ eV}) = 2.03 \text{ eV}.$$

4. As a still-lurid 80-year old in 2061, Paris Hilton drives a Hummer H34 spaceship which is 15 m long. Al Gore, long since retired but still kicking around, drives a Honda CivicsLesson spaceship which is only 3 m long. Paris and Al both fly past you while you're standing on earth, and it looks to you like they're both driving spaceships of the same length. You know that Al drives at, but not even a smidgen above, the posted speed limit of $v = 0.55c$. How fast is Paris going relative to you on earth? Relative to Al?
Answer: In their respective frames, Paris' Hummer has a proper length of $L_{1,0} = 15$ m while Al's CivicsLesson has proper length of $L_{2,0} = 3$ m. We also see a speed of $\beta_2 = 0.55$ for Al (giving $\gamma_2 = 1.20$). For both vehicles, we see the same length in our frame, or

$$\begin{aligned} L'_1 &= L'_2 \qquad \text{giving} \qquad \frac{L_{1,0}}{\gamma_1} = \frac{L_{2,0}}{\gamma_2} \\ \gamma_1 &= \frac{L_{1,0}}{L_{2,0}}\gamma_2 = \frac{15}{3} \cdot 1.20 = 6.0. \end{aligned}$$

Now from $\gamma = 1/\sqrt{1-\beta^2}$ we can find $\beta = \sqrt{1 - 1/\gamma^2}$ so Paris' speed relative to us is

$$\beta_1 = \sqrt{1 - 1/(6.0)^2} = 0.986$$

as viewed by us on the ground. We now need to shift by a velocity of $0.55c$ from our frame to Al's frame to see how Al perceives the speed of Paris' Hummer H34, so that we have $v_1 = 0.986c$ (what we see for Paris) and $v = 0.55c$ (the velocity needed to shift into Al's frame), or

$$v_2 = \frac{v_1 - v}{1 - vv_1/c^2} = \frac{0.986c - 0.55c}{1 - 0.55 \cdot 0.986} = 0.953c.$$

5. As a result of Compton scattering with a 100 keV incident photon, an electron with a momentum of 108 keV/c is created. At what angle is the Compton photon scattered?
Answer: We first want to calculate the kinetic energy of the electron from its momentum. We can do this in one of two ways:

   (a) An electron with a momentum $p = 108$ keV/c has a kinetic energy E_k of

$$\begin{aligned} E_k &= E - mc^2 = \sqrt{(mc^2)^2 + p^2c^2} - mc^2 \\ &= \sqrt{(511 \text{ keV})^2 + (108 \text{ keV})^2} - (511 \text{ keV}) = 11.2 \text{ keV}. \end{aligned}$$

   (b) One can also calculate the electron's momentum from

$$p = \gamma m v = \gamma\beta mc \qquad \Rightarrow \qquad \gamma\beta = \frac{pc}{mc^2}.$$

Now let $k \equiv pc/mc^2 = (108 \text{ keV})/(511 \text{ keV}) = 0.211$ and recall $\gamma = 1/\sqrt{1-\beta^2}$:

$$\begin{aligned} k = \frac{\beta}{\sqrt{1-\beta^2}} &\Rightarrow k^2 - k^2\beta^2 = \beta^2 \\ \beta^2(1+k^2) = k^2 &\Rightarrow \beta^2 = \frac{k^2}{1+k^2} \\ \gamma &= \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-k^2/(1+k^2)}} \\ &= \frac{1}{\sqrt{1-0.211^2/(1+0.211^2)}} = 1.022 \end{aligned}$$

Finally, $E_k = (\gamma - 1)mc^2 = (1.022 - 1)(511 \text{ keV}) = 11.2$ keV.

(c) Classically, we have

$$E_k = \frac{p^2}{2m} = \frac{(108 \text{ keV}/c)^2}{2(511 \text{ keV}/c^2)} = 11.4 \text{ keV}$$

which is only slightly off of the more correct relativistic result.

So, by either relativistic means we have determined that the kinetic energy of the scattered electron is $E_k = 11.2$ keV. Conservation of energy tells us that the energy of the scattered photon is $E_s = 100 - 11.2 = 88.8$ keV, and of course the energy of the incident photon is $E_0 = 100$ keV. We can then find the angle of the scattered photon from the Compton formula:

$$\begin{aligned} \lambda_s - \lambda_0 &= \frac{h}{mc}(1-\cos\theta) \\ \frac{mc^2}{hc}\left(\frac{hc}{E_s} - \frac{hc}{E_0}\right) &= 1 - \cos\theta \\ \theta &= \arccos\left(1 - mc^2(\frac{1}{E_s} - \frac{1}{E_0})\right) \\ &= \arccos\left(1 - (511 \text{ keV})(\frac{1}{88.8 \text{ keV}} - \frac{1}{100 \text{ keV}})\right) = 69.2^\circ \end{aligned}$$