

PHY 251, Fall 2009: Equations for Final Exam

This is the December 14, 2009 version.

	kg	MeV/c ²	amu
m_e	9.11×10^{-31}	0.510999	0.000549
m_p	1.673×10^{-27}	938.272	1.007276
m_n	1.675×10^{-27}	939.566	1.008665
amu	1.660×10^{-27}	931.494	1

1 eV = 1.602×10^{-19} Joule, $h = 6.62 \times 10^{-34}$ J·sec = 4.14×10^{-15} eV·sec. $hc = 1239.8$ eV·nm. $k_B = 1.38 \times 10^{-23}$ J/K = 8.62×10^{-5} eV/K. $c = 3.00 \times 10^8$ m/sec. $\epsilon_0 = 8.85 \times 10^{-12}$ in mks units. $\mu_B = e\hbar/(2m) = 9.274 \times 10^{-24}$ J/T. 1 Gray = 1 J/kg = 100 rad. Sievert = Gray·RBE = 100 rem. 1 Curie = 3.7×10^{10} decay/sec. $N_A = 6.02 \times 10^{23}$ atoms/mol.

Centripetal force to maintain circular motion: $\gamma m v^2 / r$. Lorentz force: $q\vec{v} \times \vec{B}$. Volume element: $r^2 dr \sin \theta d\theta d\varphi$. Constant a : $x = x_0 + v_0 t + \frac{1}{2} a t^2$, $v = v_0 + a t$, $v^2 - v_0^2 = 2a(x - x_0)$.

$$F = ma = dp/dt, E_k = \frac{1}{2} m v^2, p = m v, \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. p = \gamma m_0 v, F_{\perp} = \gamma m_0 a,$$

$$F_{\parallel} = \gamma^3 m_0 a. \nu = \frac{\nu_0}{\gamma[1 + (v/c) \cos \theta]} \text{ with } \theta = 0 \text{ for emitter moving directly away.}$$

$$\begin{aligned} x_2 &= \gamma(x_1 - v t_1) & v_{2,x} &= \frac{v_{1,x} - v}{1 - \frac{v v_{1,x}}{c^2}} & p_{x,2} &= \gamma(p_{x,1} - v(E/c^2)) \\ y_2 &= y_1 & v_{2,y} &= \frac{v_{1,y}}{\gamma \left[1 - \frac{v v_{1,x}}{c^2} \right]} & p_{y,2} &= p_{y,1} \\ z_2 &= z_1 & & & p_{z,2} &= p_{z,1} \\ t_2 &= \gamma \left(t_1 - \frac{v}{c^2} x_1 \right) & v_{2,z} &= \frac{v_{1,z}}{\gamma \left[1 - \frac{v v_{1,x}}{c^2} \right]} & E_2 &= \gamma(E - v p_x). \end{aligned}$$

$$E = E_0 + E_k = m_0 c^2 + (\gamma - 1) m_0 c^2, E^2 = E_0^2 + p^2 c^2. p = E/c. E = h\nu = hc/\lambda.$$

$$u_{\nu} d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp[h\nu/kT] - 1} d\nu \quad \lambda_{\text{peak}} = hc/(4.965 k_B T)$$

$$E_k = h\nu - \phi. \lambda = h/p, \lambda_s - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta). 2d \sin \theta = n\lambda.$$

$$r_n = \frac{n^2}{Z} a_0 \text{ with } a_0 = \frac{\epsilon_0 \hbar^2}{m \pi e^2} = 0.053 \text{ nm, and } E_n = -\frac{Z^2}{n^2} E_0 \text{ with } E_0 = \frac{m e^4}{8 \epsilon_0^2 \hbar^2} = 13.60 \text{ eV.}$$

$$m_r = \frac{m_1 m_2}{m_1 + m_2}. \frac{1}{4\pi\epsilon_0} \frac{Z e^2}{r^2} = m \frac{v^2}{r}. -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + U\psi = i\hbar \frac{\partial \psi}{\partial t} = E\psi \text{ if } U \text{ is time independent.}$$

$\Delta E \cdot \Delta t \gtrsim \hbar/2, \Delta x \cdot \Delta p \gtrsim \hbar/2$. For U constant:

$$E > U: \psi = A \sin kx + B \cos kx, k = \sqrt{2m(E - U)}/\hbar, \text{ and}$$

$$E < U: \psi = C \exp[-\alpha x], \alpha = \sqrt{2m(U - E)}/\hbar.$$

Infinite well: $E_n = n^2 \pi^2 \hbar^2 / (2mL^2)$ with $\psi = \sqrt{2/L} \sin(n\pi x/L)$

Harmonic oscillator: $E_n = (n + \frac{1}{2}) \hbar \omega, \psi_0 = A \exp[-\omega m x^2 / (2\hbar)]$.

Coulomb potential: $\psi(r, \theta, \varphi) = R_{n,\ell}(r) \Theta_{\ell, m_\ell}(\theta) \Phi_{m_\ell}(\varphi) = R_{n,\ell}(r) Y_{\ell}^{m_\ell}(\theta, \varphi)$.

$$\langle f \rangle = \int f P(x) dx. \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Energy order of shells: $1s < 2s < 2p < 3s < 3p < 4s \lesssim 3d < 4p < 5s < 4d < 5p$

$$< 6s < 4f \lesssim 5d < 6p < 7s < 6d \lesssim 5f \dots$$

$$|L| = \sqrt{\ell(\ell+1)\hbar}, L_z = m_\ell \hbar, \vec{\mu}_L = -(e/2m)\vec{L}. |S| = \sqrt{s(s+1)\hbar}, S_z = m_s \hbar, \vec{\mu}_s = -(e/m)\vec{S}.$$

$$U = m_\ell \mu_B B = 2m_s \mu_B B$$

$$n(E) dE = f(E) g(E) dE. \text{ Gibbs factor: } \exp[(N\mu - E)/(k_B T)].$$

$$f_{\text{MB}}(E) = \frac{1}{\exp[E/k_B T]}, f_{\text{FD}}(E) = \frac{1}{\exp[(E - E_F)/(k_B T)] + 1}, f_{\text{BE}}(E) = \frac{1}{\exp[E/(k_B T)] - 1}.$$

$$E_F = \frac{\hbar^2}{8m} \left(\frac{3N}{\pi V} \right)^{2/3}, C_V = \frac{9Nk_B^2}{2E_{F0}} T. \frac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3. B_{12} = B_{21}.$$

$$R = r_0 A^{1/3} \text{ with } r_0 = 1.2 \times 10^{-15} \text{ m. BE} = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N - Z)^2}{A} \text{ with}$$

$$a_1 = 15.5 \text{ MeV}, a_2 = 16.8 \text{ MeV}, a_3 = 0.72 \text{ MeV}, \text{ and } a_4 = 19 \text{ MeV. } N = N_0 \exp[-\lambda t], \text{ activity } A = \lambda N.$$

$$\begin{aligned} \int x^m e^{ax} dx &= e^{ax} \sum_0^m (-1)^r \frac{m! x^{m-r}}{(m-r)! a^{r+1}} & \int_0^\infty x^n e^{-ax} dx &= \frac{n!}{a^{n+1}} \\ \int_0^\infty x^{2n} e^{-ax^2} dx &= \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}} & \int_0^\infty x^{2n+1} e^{-ax^2} dx &= \frac{n!}{2a^{n+1}} \\ \int \sin^2(ax) dx &= \frac{x}{2} - \frac{\sin(2ax)}{4a} & \int \cos^2(ax) dx &= \frac{x}{2} + \frac{\sin(2ax)}{4a} \end{aligned}$$

$$(1+x)^n \simeq 1 + nx \text{ for } x \ll 1.$$