PHY 251, Fall 2009: Equations for Exam 2

This is the October 29, 2009 version.

Start of the periodic table: H, He, Li, Be, B, C, N, O, F, Ne, Na, Mg, Al, Si, P, S, Cl, Ar, K, Ca...

Constant acceleration $a = dv/dt$: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 - v_0^2 = 2a(x - x_0)$.

$F = ma = dp/dt$. $p = mv$ (classically) or $p = \gamma mv$ (relativistically) with $\gamma = 1/\sqrt{1-\beta^2}$.

$E_k = \frac{1}{2}mv^2$ (classically) or $E_k = (\gamma - 1)mc^2$ (relativistically).

$p = h/\lambda$, $E = hc/\lambda = h\nu$ with $hc = 1240$ eV·nm. 1 eV=1.602×10^{-19} J.

$h = 4.14 \times 10^{-15}$ eV·sec, $h = 6.63 \times 10^{-34}$ J·sec, $\hbar = h/(2\pi)$, and $c = 3.00 \times 10^8$ m/sec.

Masses: $m_e = 511$ keV/c^2, $m_p = 939$ MeV/c^2. $\Delta E\, \Delta t \geq \hbar/2$, $\Delta x\, \Delta p \geq \hbar/2$.

Bohr model: $r_n = \dfrac{n^2}{Z}a_0$ with $a_0 = \dfrac{\epsilon_0 h^2}{m\pi e^2} = 0.053$ nm,

and $E_n = -\dfrac{Z^2}{n^2}E_0$ with $E_0 = \dfrac{me^4}{8\epsilon_0^2 h^2} = 13.60$ eV.

$m_r = \dfrac{m_1 m_2}{m_1 + m_2}$. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2} = m\dfrac{v^2}{r}$.

$-\dfrac{\hbar^2}{2m}\nabla^2\psi + U\psi = i\hbar\dfrac{\partial\psi}{\partial t} = E\psi$ if U is time independent.

3D Schrödinger equation with Coulomb potential: $\psi(r,\theta,\varphi) = R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\varphi) = R_{n,\ell}(r)Y_\ell^{m_\ell}(\theta,\varphi)$.

Volume element: $r^2 dr\ \sin\theta d\theta\, d\varphi$.

For U constant:

$E > U$: $\psi = A\sin kx + B\cos kx,\ k = \sqrt{2m(E-U)}/\hbar$, and

$E < U$: $\psi = C\exp[-\alpha x],\ \alpha = \sqrt{2m(U-E)}/\hbar$.

Infinite well: $E_n = n^2\pi^2\hbar^2/(2mL^2)$ with $\psi = \sqrt{2/L}\sin(n\pi x/L)$

Harmonic oscillator: $E_n = (n + \frac{1}{2})\hbar\omega$, $\psi_0 = A\exp[-\omega m x^2/(2\hbar)]$.

$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

Energy order of shells: $1s < 2s < 2p < 3s < 3p < 4s \lesssim 3d < 4p < 5s < 4d < 5p$
$< 6s < 4f \lesssim 5d < 6p < 7s < 6d \lesssim 5f \ldots$

$|L| = \sqrt{\ell(\ell+1)}\hbar$, $L_z = m_\ell\hbar$, $\vec{\mu}_L = -(e/2m)\vec{L}$.

$|S| = \sqrt{s(s+1)}\hbar$, $S_z = m_s\hbar$, $\vec{\mu}_s = -(e/m)\vec{S}$.

$U = m_\ell\mu_B B = 2m_s\mu_B B$ with $\mu_B = e\hbar/(2m) = 9.274 \times 10^{-24}$ J/T.

$\lambda_s - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$

$2d\sin\theta = n\lambda$

$$\int x^m\, e^{ax}\, dx = e^{ax}\sum_0^m (-1)^r \frac{m!\, x^{m-r}}{(m-r)!\, a^{r+1}} \qquad \int_0^\infty x^n\, e^{-ax}\, dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n}\, e^{-ax^2}\, dx = \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}} \qquad \int_0^\infty x^{2n+1}\, e^{-ax^2}\, dx = \frac{n!}{2a^{n+1}}$$

$$\int \sin^2(ax)\, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} \qquad \int \cos^2(ax)\, dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$